

Competition for Workers and the Labor Share^{*}

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Abstract

This paper studies when labor market competition raises the aggregate labor share. Competition can reduce wage markdowns at each firm, but it also moves workers up the job ladder to more productive firms, which tend to have lower labor shares. I derive an accounting identity for the labor share that holds under any wage-setting protocol. When workers move only to more productive firms, the identity characterizes how the labor share responds to competition. I apply it to two standard protocols. With Pareto productivity, competition does not change the labor share under wage posting. In contrast, under sequential auctions, it even lowers the labor share for any bargaining power below one, because workers accept lower starting wages in return for more frequent raises. Nevertheless, labor productivity distributions in U.S. manufacturing imply that competition raises the labor share under both protocols unless bargaining power under sequential auctions is close to zero.

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1 Introduction

A common view is that workers receive a larger share of output when employers compete more for them. This view appears often in the debate on the decline of the labor share.¹ Leading explanations for the decline include a loss of worker power ([Stansbury and Summers, 2020](#)) and a rise in the monopsony power of employers ([Krueger, 2018](#)). However, competition also moves workers toward more productive firms, and these firms tend to have lower labor shares ([Autor et al., 2020](#); [Kehrig and Vincent, 2021](#)).² It is therefore not obvious whether competition raises the aggregate labor share.

This paper characterizes how labor market competition affects the aggregate labor share in the canonical job-ladder model under any wage-setting protocol in which workers move only to more productive firms, including wage posting ([Burdett and Mortensen, 1998](#)) and sequential auctions ([Postel-Vinay and Robin, 2002](#); [Cahuc et al., 2006](#)). The answer depends on the shape of the productivity distribution and, crucially, on how wages are set. Under every such protocol, competition moves workers up the ladder in the same way, and wage setting matters only through how it divides rents between firms and workers. With Pareto productivity, for example, competition does not change the labor share under wage posting ([Gouin-Bonenfant, 2022](#); [Lindenlaub et al., 2025](#)). In contrast, under sequential auctions, it even lowers the labor share for any bargaining power below one. Nevertheless, for the distributions of labor productivity in U.S. manufacturing, whose upper tails are closer to lognormal than to Pareto, competition raises the labor share under both protocols, unless bargaining power under sequential auctions is close to zero.

I start by deriving a general accounting identity for the aggregate labor share. The identity does not require that workers move only to more productive firms: it holds for any allocation of workers across firms, as long as the least productive active firm earns no rent. It takes the following simple form:

$$1 - \text{Labor Share} = \mathbb{E}^Y [\tau(z) \sigma(z)].$$

The profit multiplier $\tau(z)$ compares two firms whose productivities differ slightly: each worker of the more productive firm produces a little more, and $\tau(z)$ is the difference in

¹See [Elsby et al. \(2013\)](#) and [Karabarbounis and Neiman \(2014\)](#) for the decline of the labor share.

²[Seegmiller \(2025\)](#) uses data on U.S. public firms and finds that more productive firms pay a smaller fraction of the marginal product of labor.

their profits per unit of this extra output. The tail thickness $\sigma(z)$ is the number of firms more productive than z relative to the number of firms close to z , with firms weighted by their vacancies.³ The operator $\mathbb{E}^Y[\cdot]$ is the mean with output weights. Importantly, for a given allocation of workers, the profit multiplier $\tau(z)$ is the only object that depends on how wages are set, and it equals one under wage posting. The tail thickness $\sigma(z)$ depends only on the productivity distribution, and the allocation of workers enters only through the output weights. Behind the identity is a view of productivity as a ladder. The profit of a firm is the sum of the differences in profit between adjacent rungs below it, and the difference at each rung is part of the profit of every firm above it. Summed over all the firms above the rung, this difference is the rent generated at that rung.

The main result expresses the response of the labor share to competition, measured by net upward mobility, through the same objects. For a given distribution of vacancies across firms, the response consists of a reallocation term and a wage-setting term. The reallocation term compares where rents are generated with where output is produced. The wage-setting term captures how competition changes the profit multiplier $\tau(z)$ at a given allocation. It is zero under wage posting, and under sequential auctions, it reflects the lower starting wages that workers accept in return for more frequent raises.

The identity shows when competition raises the labor share. Competition moves workers, and with them output, toward more productive firms. Under wage posting, the ratio of the rents generated at each rung to the output produced there equals the tail thickness $\sigma(z)$. Competition does not change $\sigma(z)$, which depends only on the distribution of vacancies. Both the fall in the markdown that competition causes at every firm and the reallocation of workers therefore appear in the identity only as a shift of output across rungs. The labor share rises if the tail thickness falls with productivity, because output moves to rungs that generate little rent relative to their output. With Pareto productivity, the tail thickness does not depend on z , and moving output across firms does not change the labor share. The neutrality continues to hold when firms choose their vacancies. The same logic, including the neutrality under Pareto productivity, governs the effect of tougher competition in product markets with variable markups (Arkolakis et al., 2019; Autor et al., 2020).

In contrast to wage posting, competition under sequential auctions also changes the path of wages within a job. More competition makes the raises that future outside offers bring

³In other words, it is the inverse of the local Pareto exponent of the underlying (vacancy-weighted) productivity distribution.

more valuable, and workers accept lower starting wages in return. In the identity, this raises the profit multiplier $\tau(z)$. With Pareto productivity, this effect outweighs the reallocation of workers, and competition lowers the labor share even though workers climb further up the ladder and receive more raises. The neutrality under Pareto productivity is thus specific to wage posting. Like wage posting, static product-market models have no counterpart to this effect, because it comes from the *path* of wages within a job.

I then measure the objects in the identity and in the response formula. They involve the productivity distribution across vacancies, which is not observed. The steady state of the job ladder connects it to the distribution of productivity across employed workers, which establishment-level data measure under some conditions. Given the wage-setting protocol, this distribution and the degree of competition determine the labor share and every term of the response formula. I use the Dispersion Statistics on Productivity (DiSP) of the Bureau of Labor Statistics and the Census Bureau. With these data, I show that the tail thickness of labor productivity in U.S. manufacturing falls with productivity. As a result, more competition raises the labor share in almost all industry-years, unless bargaining power under sequential auctions is close to zero. The common view is consistent with the data. The reason is the shape of the productivity distribution, not only the fall in markdowns.

Finally, I evaluate a large historical change in competition. [Engbom et al. \(2026\)](#) estimate that employed workers received better-paying outside offers about half as often in the 2010s as in the 1980s, and they relate the decline to employer concentration and noncompete agreements. For a given distribution of vacancies and with the degree of competition implied by the flows of U.S. workers, this decline lowers the labor share of manufacturing by 1.8 percentage points under wage posting and 0.9 points under sequential auctions with equal bargaining power. The effect is smaller under sequential auctions because workers who expect fewer raises are hired at higher starting wages. When workers have no bargaining power, higher starting wages more than offset the reallocation of workers down the ladder, and the decline in offers raises the labor share.

Related literature. This paper is most closely related to work on the labor share in frictional labor markets. [Gouin-Bonenfant \(2022\)](#) links productivity dispersion to the labor share in a wage-posting model with firm entry, exit, and changes in productivity, an environment richer than mine. In the Burdett–Mortensen special case of his model, in which productivity differences are permanent, his formula for the labor share with Pareto

productivity does not depend on competition. Using a distribution that fits the data better, he shows that rising dispersion lowers the labor share, mainly through a reallocation of value added toward firms with low labor shares. [Mangin \(2017\)](#) studies a static model in which firms compete in second-price auctions for unemployed workers. She shows that the productivity rents of firms, relative to output, equal the output-weighted mean of the inverse of the local Pareto exponent of productivity, which she calls the generalized hazard rate. This is the static counterpart of my identity under wage posting. [Mangin \(2017\)](#) also shows that the labor share rises with the expected number of competing firms when this exponent is increasing, and that productivity rents are a constant share of output under Pareto productivity. Relative to these papers, I characterize the response to competition on a job ladder, for any productivity distribution and under any wage-setting protocol in which workers move only to more productive firms. Because employed workers keep receiving outside offers, competition can also change how wages evolve within a job. Under sequential auctions, this breaks the Pareto neutrality that holds under wage posting and in the static model.

The paper also relates to work on the decline of the labor share. Much of the decline comes from the reallocation of value added to producers with low labor shares ([Autor et al., 2020](#); [Kehrig and Vincent, 2021](#)). [Kehrig and Vincent \(2021\)](#) find that, in U.S. manufacturing, this reallocation is associated with transient increases in value added per worker and product prices rather than declines in wages. They interpret it as the result of shocks to product demand. In both studies, sales and value added are reallocated much more than workers. Using stock returns as shifts in labor demand, [Seegmiller \(2025\)](#) finds that more productive U.S. public firms face less elastic labor supply and pay a smaller fraction of the marginal product of labor, and that the fall in these elasticities accounts for a large part of the decline in their labor share at fixed employment. I take the distribution of productivity across firms as given, whatever its source, and ask how competition for workers, which moves workers toward producers with low labor shares, affects the labor share.

Finally, the paper relates to work on the decline of job mobility and on employer power. [Engbom et al. \(2026\)](#) show in a model of wage posting that the decline in outside offers to employed workers has slowed wage growth, mostly because firms that face less competition post lower wages. Recent work on labor market power studies how employer concentration

lowers wages (Berger et al., 2024; Jarosch et al., 2024). In my model, firms are atomistic, and competition comes from the outside offers that employed workers receive. I show that wage setting determines the size of the effect of the decline in these offers on the labor share, and even its sign.

2 Environment

This section describes a stationary labor market with on-the-job search and heterogeneous firms, which is common to the two wage-setting protocols studied in the paper.

2.1 Agents and Meetings

Time is continuous, and the horizon is infinite. Workers and firms discount future payoffs at rate r . There is a unit mass of risk-neutral workers and a fixed mass of firms, normalized to one.⁴ Firms differ in productivity z , which is distributed according to Ψ with support $[z_0, \bar{z}]$, where $z_0 > 0$ and $\bar{z}$ may be infinite. Technology is linear: a worker employed by a firm with productivity z produces z units of output per unit of time. An unemployed worker enjoys a flow value b .

Workers and firms meet through vacancies. A firm with productivity z posts $v(z)$ vacancies. Let $V \equiv \int v(z) d\Psi(z)$ denote the total number of vacancies and

$$\hat{F}(z) \equiv \frac{1}{V} \int_{z_0}^z v(x) d\Psi(x) \quad (1)$$

the vacancy-weighted distribution of productivity, with density $\hat{f}$.

Search is random. An unemployed worker meets a vacancy at rate $\lambda^U > 0$, and an employed worker meets a vacancy at rate $\lambda^E > 0$. A vacancy meets a worker at rate $\lambda^F > 0$. The worker it meets is drawn from the pool of searchers in proportion to search intensity: each unemployed worker receives weight λ^U and each employed worker weight λ^E . Employment relationships end at rate $\delta > 0$, after which the worker returns to unemployment. I take the meeting rates as given and, until Section 6, also the distribution $\hat{F}$.

⁴Since I assume that technology is linear, the mass of firms only scales the size of each firm and does not affect the labor share.

I study steady states in the limit in which the discount rate r goes to zero. In this limit, firms maximize steady-state profit flows, as in [Burdett and Mortensen \(1998\)](#), and worker values are evaluated as limits of discounted values.⁵

2.2 Wage-Setting Protocols

Throughout the paper, I study the following two widely used wage-setting protocols.

Wage posting. Each firm commits to one wage for all of its workers, as in [Burdett and Mortensen \(1998\)](#). An employed worker compares the wage posted by the firm that the worker meets with the current wage. A firm with productivity z chooses its wage $w(z)$ to maximize its profit flow per vacancy.

Sequential auctions. Wages are specific to each worker and are renegotiated when outside offers arrive, as in [Postel-Vinay and Robin \(2002\)](#) and [Cahuc et al. \(2006\)](#). When an employed worker meets another firm, the current employer and the other firm compete for the worker. The worker ends up at the more productive firm and receives the value of the maximal offer of the less productive firm plus a share $\beta \in [0, 1)$ of the difference between the two match surpluses, if this exceeds the value of the current wage. An unemployed worker hired by a firm receives the share β of the match surplus.⁶

I maintain the following assumption.

Assumption 1.

- (i) *Efficient mobility.* An employed worker who meets an outside firm moves if and only if the new firm is more productive than the current employer. An unemployed worker accepts every offer from an active firm, where a firm is active if its productivity is not smaller than the reservation level $\underline{w}$.
- (ii) *No rent at the bottom.* The support of Ψ extends down to the reservation level, $z_0 \leq \underline{w}$.

⁵Proposition [D.1](#) in Supplemental Appendix [D.5](#) treats sequential auctions with $r > 0$, and Section [4.2](#) discusses what changes.

⁶The case in which workers have no bargaining power, $\beta = 0$, is the protocol of [Postel-Vinay and Robin \(2002\)](#).

Part (i) is a property of both protocols rather than a restriction. Under sequential auctions, it holds by construction, and under wage posting, it holds in equilibrium because the wage $w(z)$ is strictly increasing in productivity (Bontemps et al., 2000). The reservation level $\underline{w}$ is the reservation wage under wage posting and the productivity at which the match surplus vanishes under sequential auctions. Firms with productivity below $\underline{w}$ cannot hire at a profit, are inactive, and post no vacancies, so that $\hat{F}$ puts no weight below $\underline{w}$. By part (ii), the least productive active firm has productivity $\underline{z} = \underline{w}$ and earns no rent. I assume that the density $\hat{f}$ is positive on $(\underline{z}, \bar{z})$.

2.3 Steady-State Allocation

A worker employed by a firm with productivity z leaves the firm at rate

$$T(z) \equiv \delta + \lambda^E (1 - \hat{F}(z)). \quad (2)$$

The employed worker moves to unemployment at an exogenous separation rate δ and leaves for a more productive firm at rate $\lambda^E (1 - \hat{F}(z))$. Let u denote the unemployment rate, and let $\hat{G}(z)$ denote the probability that a vacancy meets a worker who is either unemployed or employed by a firm with productivity at most z .

Lemma 1 (Allocation). *Under Assumption 1(i), in steady state,*

$$u = \frac{\delta}{\delta + \lambda^U}, \quad \hat{G}(z) = \frac{\delta}{T(z)}, \quad (3)$$

and a vacancy of a firm with productivity z hires $h(z) = \lambda^F \hat{G}(z)$ workers per unit of time. Employment per vacancy, $\ell(z)$, is given by

$$\ell(z) = \frac{h(z)}{T(z)} = \frac{\lambda^F \hat{G}(z)^2}{\delta} = \frac{\lambda^F}{\delta} \left[1 + \kappa (1 - \hat{F}(z)) \right]^{-2}, \quad \kappa \equiv \frac{\lambda^E}{\delta}. \quad (4)$$

Proof. The equations are the steady-state flow conditions of Burdett and Mortensen (1998), with productivity in place of wages. Supplemental Appendix D.1 derives them. $\square$

Two features of Lemma 1 are used throughout the paper. First, the allocation does not depend on how the rents of a match are divided: it is the same under both protocols. Second, up to the factor λ^F/δ , employment per vacancy, $\ell(z)$, depends on the meeting and

separation rates only through the single parameter κ , the expected number of outside offers that an employed worker receives before returning to unemployment. I refer to κ as the degree of labor market competition. The rate λ^U at which unemployed workers find jobs affects the unemployment rate, but not the distribution of employment across firms.

Let $L(z)$ denote the share of employed workers who work at firms with productivity at most z . Workers enter this group of firms only from unemployment, at rate $\lambda^U u \hat{F}(z)$, and each worker leaves it at rate $T(z)$, through a separation to unemployment or a move to a firm with productivity above z . In steady state, the two flows are equal, $\lambda^U u \hat{F}(z) = T(z) (1 - u) L(z)$. Since $\lambda^U u = \delta(1 - u)$ by Lemma 1,

$$L(z) = \frac{\hat{F}(z)}{1 + \kappa(1 - \hat{F}(z))}, \quad \text{equivalently} \quad \hat{F}(z) = \frac{(1 + \kappa) L(z)}{1 + \kappa L(z)}. \quad (5)$$

This equation provides the connection between the vacancy-weighted productivity distribution and the employment-weighted productivity distribution. It plays a central role in Section 5, where I use data on $L(z)$ to infer $\hat{F}(z)$.

A firm with productivity z employs $v(z) \ell(z)$ workers, and aggregate output is therefore given by

$$Y = \int_{\underline{z}}^{\bar{z}} z \ell(z) v(z) d\Psi(z) = V \int_{\underline{z}}^{\bar{z}} z \ell(z) d\hat{F}(z). \quad (6)$$

I assume that output is finite.

3 Accounting for the Response to Competition

This section derives an accounting identity for the aggregate labor share that uses only part (ii) of Assumption 1, and thus holds under any wage-setting protocol and any allocation of workers. Under both parts of the assumption, it then expresses the response of the labor share to competition through the objects of the identity.

3.1 An Accounting Identity

Let $\pi(z)$ denote the steady-state per-vacancy profit flow of a firm with productivity z : the output of the workers employed through one vacancy minus their wages. I assume that π is absolutely continuous on $[\underline{z}, \bar{z}]$ and that profits are finite. If π is not monotone, I also

assume that $\int_{\underline{z}}^{\bar{z}} |\pi'(z)|(1 - \hat{F}(z)) dz$ is finite. Aggregate profits are $V \int_{\underline{z}}^{\bar{z}} \pi(z) d\hat{F}(z)$, and the aggregate labor share is

$$\text{LS} \equiv 1 - \frac{V \int_{\underline{z}}^{\bar{z}} \pi(z) d\hat{F}(z)}{Y}. \quad (7)$$

One minus the labor share is thus the ratio of two aggregates, profits and output. I write both as sums over the rungs of the productivity ladder, which shows how differently they weight each rung.

I start with profits. By Assumption 1(ii), the least productive active firm earns no profit, $\pi(\underline{z}) = 0$.⁷ A firm's profit is then built up by accumulating the profit increments associated with all the rungs of the productivity ladder below it,

$$\pi(z) = \int_{\underline{z}}^z \pi'(x) dx. \quad (8)$$

I divide the increment by employment per vacancy and call it the *profit multiplier*, defined wherever $\pi(z)$ is differentiable,

$$\tau(z) \equiv \frac{\pi'(z)}{\ell(z)}. \quad (9)$$

The profit multiplier compares firms on adjacent rungs: each worker of a firm that is dz more productive produces dz more, and $\tau(z)$ is the ratio of the difference in profits per vacancy, $\pi'(z) dz$, to this extra output, $\ell(z) dz$. Under wage posting, it equals one (Section 4.1).

To aggregate, I reverse the order of accounting. Rather than starting with each firm and adding up the rungs below it, I start with each rung and ask how many vacancies lie above it. The difference in profit across a small step dz at productivity z , $\pi'(z) dz = \tau(z)\ell(z) dz$, is counted in the profit of every vacancy above z , a fraction $1 - \hat{F}(z)$ of all vacancies. Output, in contrast, is produced at each rung only by the vacancies at that rung, a fraction $\hat{f}(z) dz$, each of which employs $\ell(z)$ workers who produce z . Per vacancy, aggregate profits and

⁷Its productivity equals the reservation level (Section 2.2). If part (ii) failed and the support of Ψ started above the reservation level, the least productive active firm would earn a rent $\pi(\underline{z}) > 0$, equal to $\tau(\underline{z})(\underline{z} - \underline{w})\ell(\underline{z})$ under the two protocols of Section 4, and one minus the labor share would include the additional term $\pi(\underline{z})/\int_{\underline{z}}^{\bar{z}} z\ell(z) d\hat{F}(z)$. This rent is generated at the bottom of the ladder, from which competition moves employment away, and it makes a rise in the labor share with competition more likely.

aggregate output are therefore

$$\text{Profits: } \int_{\underline{z}}^{\bar{z}} \pi(z) d\hat{F}(z) = \int_{\underline{z}}^{\bar{z}} f^{\mathcal{R}}(z) dz, \quad f^{\mathcal{R}}(z) \equiv \tau(z) \ell(z) (1 - \hat{F}(z)), \quad (10)$$

$$\text{Output: } \int_{\underline{z}}^{\bar{z}} z \ell(z) d\hat{F}(z) = \int_{\underline{z}}^{\bar{z}} f^Y(z) dz, \quad f^Y(z) \equiv z \ell(z) \hat{f}(z). \quad (11)$$

I call $f^{\mathcal{R}}$ the density of rents and f^Y the density of output.⁸ The density of rents is not the distribution of profits across firms. It assigns to rung z the difference in profit at z summed over all firms above z , and I say that rents are generated at z in this sense.

The two densities share employment per vacancy, $\ell(z)$, and differ in the other two factors. Per worker at rung z , the density of rents counts $\tau(z)$ and the density of output counts z . The rents generated at z are counted in the profits of the vacancies above z , whereas the output produced at z comes from the vacancies at z . The ratio of the two densities is

$$\frac{f^{\mathcal{R}}(z)}{f^Y(z)} = \tau(z) \frac{1 - \hat{F}(z)}{z \hat{f}(z)} = \tau(z) \sigma(z), \quad (12)$$

where

$$\sigma(z) \equiv \frac{1 - \hat{F}(z)}{z \hat{f}(z)} = \frac{1}{\xi(z)}, \quad \xi(z) \equiv -\frac{d \log(1 - \hat{F}(z))}{d \log z}, \quad (13)$$

is the *tail thickness*, the local inverse Pareto exponent of the vacancy-weighted distribution. The local Pareto exponent $\xi(z)$ is the elasticity with which the share of more productive vacancies falls as productivity rises. If the productivity distribution $\hat{F}(z)$ is Pareto with exponent ξ , then the tail thickness $\sigma(z) = 1/\xi$ is constant. It falls to zero at the top of a distribution with bounded support and a positive density at the top. It falls toward zero in the upper tail of a lognormal distribution, more slowly than with bounded support.

Let $\mathbb{E}^Y[\cdot]$ denote the mean with output weights: for a function $x(z)$,

$$\mathbb{E}^Y[x(z)] \equiv \frac{\int_{\underline{z}}^{\bar{z}} x(z) f^Y(z) dz}{\int_{\underline{z}}^{\bar{z}} f^Y(z) dz}. \quad (14)$$

Dividing equation (10) by equation (11) and using equation (12) gives the following identity.

⁸Neither is a probability density: they integrate to profits and output per vacancy, not to one.

Lemma 2 (Labor share accounting). *Under Assumption 1(ii),*

$$1 - \text{LS} = \mathbb{E}^Y [\tau(z) \sigma(z)]. \quad (15)$$

Proof. See Appendix A.1. □

In words, the profit share is an output-weighted average of the product of the profit multiplier $\tau(z)$ and the tail thickness $\sigma(z)$. Each of the three objects plays one role. The tail thickness depends only on the productivity distribution, and, for a given allocation of workers, the profit multiplier is the only object that depends on how wages are set. The identity does not use part (i) of Assumption 1. Because employment per vacancy cancels from the ratio of the two densities, the allocation of workers, whatever it is, enters only through the output weights, that is, through where along the ladder output is produced. Relative to its output, a rung contributes more to the profit share when profits rise sharply there per unit of extra output, so that $\tau(z)$ is large. It also contributes more when there are many vacancies further up the ladder relative to those near it, so that $\sigma(z)$ is large. Sections 4.1 and 4.2 derive the profit multiplier under the two wage-setting protocols.

3.2 The Response to Competition

This subsection asks how the labor share responds to the degree of labor market competition $\kappa = \lambda^E / \delta$. Competition rises when employed workers receive offers more often or when employment relationships end less often. Until Section 6, I hold fixed the vacancy-weighted distribution $\hat{F}(z)$ and the reservation level $\underline{w}$, which equals the lowest active productivity $\underline{z}$. Section 6 lets vacancies respond.

By Lemma 1, competition changes the allocation through the elasticity of employment per vacancy, $\ell(z)$ in equation (4), relative to that of the least productive active firm,

$$\epsilon_\ell^\kappa(z) \equiv \frac{\partial \log [\ell(z) / \ell(\underline{z})]}{\partial \log \kappa} = \frac{2\kappa}{1 + \kappa} \frac{\hat{F}(z)}{1 + \kappa(1 - \hat{F}(z))} = \frac{2\kappa}{1 + \kappa} L(z) \geq 0. \quad (16)$$

Only relative employment matters for the labor share, because a factor of $\ell(z)$ that is common to all firms, such as λ^F / δ in equation (4), cancels. The elasticity is zero at the bottom and strictly increasing in z : more competition moves employment toward more productive firms. The last equality uses equation (5): the elasticity is proportional to the

share of employed workers who work at firms with productivity at most z . Competition can also change the profit multiplier, through

$$\epsilon_\tau^\kappa(z) \equiv \frac{\partial \log \tau(z)}{\partial \log \kappa}. \quad (17)$$

Under wage posting, $\tau(z) = 1$ and $\epsilon_\tau^\kappa(z) = 0$. Let $\mathcal{R}$ denote the measure of rents, with density $f^\mathcal{R}$ in equation (10), and let $\mathbb{E}^\mathcal{R}$ denote the mean with respect to $\mathcal{R}$. The following proposition shows how the aggregate labor share responds to labor market competition.

Proposition 1 (Impact of competition). *Suppose that Assumption 1 holds and that $\tau(z)$ is positive, depends on the meeting and separation rates only through κ , and is differentiable in κ , with $|\epsilon_\tau^\kappa(z)| \leq M$ for some constant M , all z , and all κ in a neighborhood. Then, at a fixed $\hat{F}$ and $\underline{w}$,*

$$\begin{aligned} \frac{d\text{LS}}{d \log \kappa} &= (1 - \text{LS}) \left[\underbrace{\mathbb{E}^Y[\epsilon_\ell^\kappa(z)] - \mathbb{E}^\mathcal{R}[\epsilon_\ell^\kappa(z)]}_{\text{reallocation}} - \underbrace{\mathbb{E}^\mathcal{R}[\epsilon_\tau^\kappa(z)]}_{\text{wage setting}} \right] \\ &= (1 - \text{LS}) \left[\frac{2\kappa}{1 + \kappa} \left(\mathbb{E}^Y[L(z)] - \mathbb{E}^\mathcal{R}[L(z)] \right) - \mathbb{E}^\mathcal{R}[\epsilon_\tau^\kappa(z)] \right]. \end{aligned} \quad (18)$$

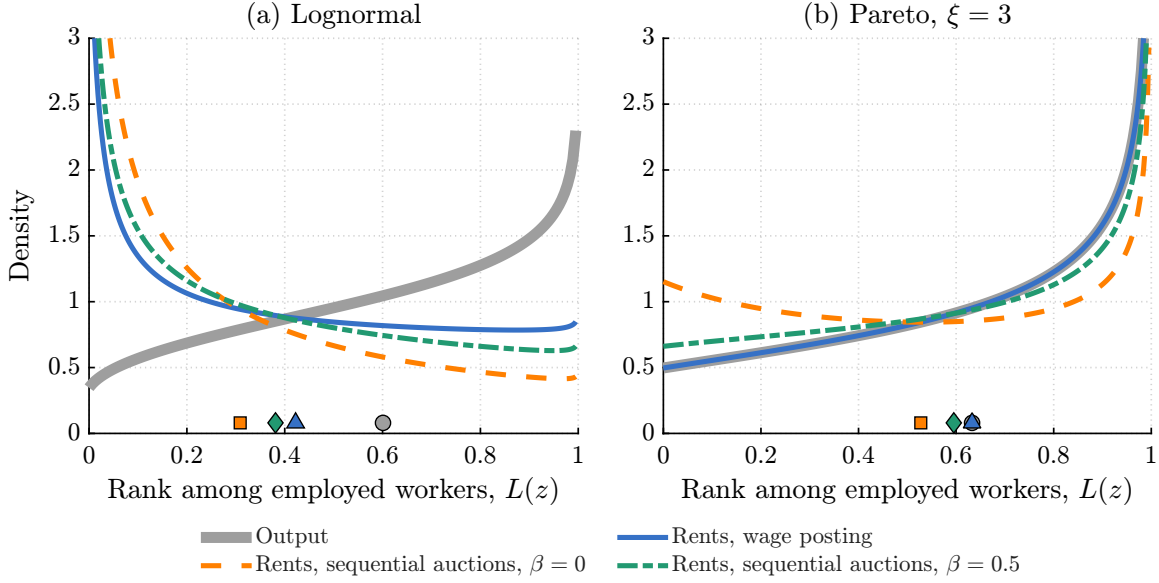
Proof. See Appendix A.2. □

The first term compares where output is produced with where rents are generated. As the second line of equation (18) shows, the term is positive exactly when the mean of the rank $L(z)$ of the firm among employed workers is higher under the output weights than under the rent weights. In other words, rents are generated, on average, at lower rungs of the ladder than output is produced. Then, through reallocation, competition raises the labor share. This is the case, for example, when the output weights f^Y strictly first-order stochastically dominate the rent weights $f^\mathcal{R}$.

By equation (12), rents are generated at lower productivity than output is produced when the ratio $\tau(z)\sigma(z)$ falls with z . The ratio falls, for example, when $\sigma(z)$ falls because the upper tail of the productivity distribution is thin, and $\tau(z)$ is constant. It also falls when $\tau(z)$ falls, even if $\sigma(z)$ is constant.

The second term captures how competition changes the division of rents through wage setting, holding the allocation fixed. It is absent under wage posting, where $\tau(z) = 1$, and Section 4.2 studies it under sequential auctions.

Figure 1: Where Rents Are Generated and Where Output Is Produced



Notes: The densities of output and of the rent measure $\mathcal{R}$ with respect to the rank of the firm among employed workers, $L(z)$ in equation (5), with $\kappa = 2.5$ and $\underline{z} = 0.4$. In panel (a), $\hat{F}$ is lognormal with log-mean $\log 0.8$ and log-standard deviation 0.4, truncated below at $\underline{z}$; in panel (b), it is Pareto with $\xi = 3$ on $[\underline{z}, \infty)$. Each density integrates to one. The markers on the horizontal axis show the mean ranks. In panel (b), the density of rents under wage posting coincides with the density of output. The curves are not drawn above a density of 3 or for the top 0.5 percent of workers.

Graphical example. Figure 1 plots the densities of rents and of output with respect to the rank of the firm among employed workers, for a lognormal and a Pareto distribution of productivity and under both protocols. Section 4.2 derives the profit multiplier under sequential auctions. With the lognormal distribution (panel a), rents are generated on lower rungs of the ladder than output is produced. Few firms are above a rung near the top, and the rung generates little rent relative to the output produced there. With the Pareto distribution (panel b), the two distributions coincide under wage posting, and rents are generated on lower rungs under sequential auctions only because $\tau(z)$ falls along the ladder. The markers on the horizontal axis show the two mean ranks in the second line of equation (18).

3.3 What the Response Depends On

Proposition 1 holds, at a fixed vacancy distribution $\hat{F}$, under any wage-setting protocol that satisfies Assumption 1, in which π is absolutely continuous, and in which $\tau(z)$ is positive, depends on the meeting and separation rates only through κ , and is differentiable in κ with

a bounded elasticity. Part (i) of Assumption 1 enters through the allocation: it makes κ the only rate that matters and gives the elasticity of employment in equation (16). Wage setting affects the response only through the profit multiplier and its elasticity, because the allocation, and with it the elasticity of employment, is the same under every protocol that satisfies Assumption 1.

By equation (12) and Lemma 2, $\mathbb{E}^R[x(z)] = \mathbb{E}^Y[\tau(z)\sigma(z)x(z)]/(1 - \text{LS})$ for any function x , and equation (18) becomes

$$\frac{d\text{LS}}{d \log \kappa} = -\text{Cov}^Y(\tau(z)\sigma(z), \epsilon_\ell^\kappa(z)) - \mathbb{E}^Y[\tau(z)\sigma(z) \epsilon_\tau^\kappa(z)], \quad (19)$$

where Cov^Y is the covariance under the output weights. The reallocation term is minus the covariance between $\tau(z)\sigma(z) = f^R(z)/f^Y(z)$, the rents generated at a rung relative to the output produced there, and the elasticity of employment $\epsilon_\ell^\kappa(z)$ at that rung. Under wage posting, $\tau(z) = 1$, and the response is $d\text{LS}/d \log \kappa = -\text{Cov}^Y(\sigma(z), \epsilon_\ell^\kappa(z))$ (Section 4.1).

Equation (19) shows that the level of the labor share and its response to competition through reallocation depend on different features of the same function $\tau(z)\sigma(z)$. The level depends on its mean: by Lemma 2, one minus the labor share is the mean of $\tau(z)\sigma(z)$ under the output weights. The response through reallocation depends on its covariance with the elasticity of employment, which rises along the ladder, and hence on whether $\tau(z)\sigma(z)$ falls or rises along the ladder, not on its level. Two economies with the same labor share can therefore respond to competition in opposite directions, and reallocation leaves the labor share unchanged when $\tau(z)\sigma(z)$ is constant, as with Pareto productivity under wage posting.

Proposition 1 gives the first-order response, and Section 5.5 evaluates a large change. Holding $\hat{F}$ fixed amounts to assuming that vacancies do not respond to competition. It also holds fixed the reservation level $\underline{w} = \underline{z}$, which determines the set of active firms. The reservation level does not depend on κ when $\lambda^U = \lambda^E$, as in the Burdett–Mortensen benchmark of Gouin-Bonenfant (2022), and under sequential auctions also when $\beta = 0$. Section 6 shows that when vacancies respond, the Pareto neutrality under wage posting survives, and it discusses the evidence on how strongly vacancies respond.

3.4 Non-Wage Amenities

Part (i) of Assumption 1 implies that every job-to-job move is a move to a more productive firm, and under wage posting (Section 4.1), every such move raises the wage. In the data, however, a sizable share of job-to-job moves come with a wage cut. Jolivet et al. (2006) and Tjaden and Wellschmied (2014) attribute many of them to involuntary moves down the ladder, which part (i) excludes. Two explanations are consistent with part (i). Under sequential auctions (Section 4.2), a worker accepts a lower wage at a more productive firm in return for future raises (Postel-Vinay and Robin, 2002). Alternatively, firms also differ in non-wage amenities, for which workers give up part of their wages (Sorkin, 2018; Morchio and Moser, 2026).

Amenities fit the analysis of this section if all workers value them in the same way and their value adds one for one to the wage, as in the hedonic search model of Hwang et al. (1998). Suppose that the jobs of a firm also provide an amenity that every worker values at a per unit of time, where a can differ across firms. A worker then values a job by the wage plus the amenity, and a firm earns $z - w = \hat{z} - (w + a)$ per worker, where $\hat{z} \equiv z + a$ is the output of a job plus its amenity. Under either protocol, the labor market is therefore the one of Section 2 with productivity $\hat{z}$ and wages $w + a$. Under sequential auctions, for example, the joint value of a match depends only on $\hat{z}$, and the firm with the higher $\hat{z}$ wins the worker. Assumption 1, Lemma 2, and Proposition 1 then apply with $\hat{z}$ in place of z . They describe the share of wages and amenities in output plus amenities. Because profits are the same, one minus the labor share of output equals $(1 - s)(1 + A/Y)$, where s is this share and A/Y is the value of amenities relative to output. Morchio and Moser (2026) use the same reduction of (z, a) to $\hat{z}$, which they call composite productivity, for each worker type in an estimated job ladder model.

If workers instead have idiosyncratic preferences over firms (Lamadon et al., 2022), they no longer rank firms in the same way, and part (i) of Assumption 1 fails. Lemma 2 still holds as long as firms differ only in productivity and the least productive active firm earns no rent, because it does not use part (i).

4 Wage Posting and Sequential Auctions

This section evaluates Lemma 2 and Proposition 1 under the two protocols of Section 2.2. For a given allocation and productivity distribution, the protocols differ only in the profit multiplier $\tau(z)$, which turns out to equal one under wage posting and fall along the ladder under sequential auctions.⁹ Section 4.3 compares the results under wage posting with those for product markets.

4.1 Wage Posting

Proposition 2 (Wage posting). *Under wage posting:*

- (i) *The wage $w(z)$ is continuous and strictly increasing on $[\underline{z}, \bar{z}]$, and part (i) of Assumption 1 holds. The least productive active firm pays the reservation wage, $w(\underline{z}) = \underline{w}$, where*

$$\underline{w} - b = (\lambda^U - \lambda^E) \int_{\underline{w}}^{w(\bar{z})} \frac{1 - F(w)}{\delta + \lambda^E(1 - F(w))} dw \quad (20)$$

and F is the distribution of posted wages across vacancies, $F(w(z)) = \hat{F}(z)$.

- (ii) *The profit multiplier is $\tau(z) = 1$ for all z , and the least productive active firm earns $\pi(\underline{z}) = 0$. Equivalently, the rent per worker $z - w(z)$ satisfies*

$$(z - w(z)) \ell(z) = \int_{\underline{z}}^z \ell(x) dx. \quad (21)$$

- (iii) *The labor share satisfies*

$$1 - \text{LS} = \mathbb{E}^Y[\sigma(z)]. \quad (22)$$

Proof. See Appendix A.3. □

⁹Other wage-setting protocols fit the same framework. Gottfries (2026) obtains continuously renegotiated Nash bargaining as the limit of bargaining with increasingly frequent renegotiation. Under this protocol, workers receive the share β of the match surplus $S(z)$, which differs from the surplus under sequential auctions in Section 4.2, and Assumption 1(i) holds. The profit multiplier is $\tau(z) = (1 - \beta)[1 + (2 - \beta)\lambda^E \hat{f}(z)S(z)]$ (Supplemental Appendix D.2). Unlike under the two protocols studied here, $\tau(z)$ then depends on the density of vacancies and on the level of the surplus, not only on the rank of the firm. With posted wage-tenure contracts (Burdett and Coles, 2003, 2010), $\tau(z) = 1$ by the envelope theorem, but part (i) of Assumption 1 fails, because workers with long tenure turn down offers from more productive firms.

Part (i) collects standard properties of the Burdett–Mortensen equilibrium with heterogeneous firms (Burdett and Mortensen, 1998; Bontemps et al., 2000; Mortensen, 2003), stated in the limit $r \rightarrow 0$. Part (ii) follows from the envelope theorem. A firm can always post the wage of a slightly less productive firm, attract the same workers, and earn the difference in productivity on each of them. In equilibrium, the value of this option determines the rent: an increase in the firm’s own productivity raises profits per vacancy by the firm’s employment per vacancy, $\pi'(z) = \ell(z)$. Equation (21) is the wage equation of Bontemps et al. (2000).

Corollary 1 (Pareto productivity). *Suppose that $\hat{F}(z) = 1 - (z/\underline{z})^{-\xi}$ on $[\underline{z}, \infty)$ with $\xi > 1$. Under wage posting, $LS = 1 - 1/\xi$ for all λ^U , λ^E , and δ .*

Proof. Under Pareto productivity, $\sigma(z) = 1/\xi$ for all z , and output is finite because ℓ is bounded and $\xi > 1$. Equation (22) then gives $1 - LS = 1/\xi$. $\square$

Gouin-Bonenfant (2022) derives the formula $1 - \alpha - 1/\xi_a$ for the labor share of value added in a Burdett–Mortensen model with capital and no discounting (Table I and Online Appendix B.1 of that paper). In this formula, α is the capital elasticity of output, and ξ_a is the Pareto exponent of total factor productivity. With capital as in Section 5.1, net output per worker has the Pareto exponent $\xi = (1 - \alpha)\xi_a$, and Corollary 1 gives the same formula.¹⁰ Lemma 2 and Proposition 2 show why frictions drop out: with Pareto productivity, $\sigma(z)$ is constant, and the output weights, which are the only channel through which frictions operate, become irrelevant. The labor share does not converge to one as competition becomes perfect. As $\kappa \rightarrow \infty$, employment moves to ever more productive firms, whose tail thickness remains $1/\xi$. With bounded support, instead, $\sigma(\bar{z}) = 0$, and the labor share converges to one.

With $\tau(z) = 1$, the wage-setting term of Proposition 1 vanishes, and the response of the labor share to competition consists of the reallocation term alone.

Proposition 3 (Competition under wage posting). *Under wage posting:*

- (i) *The rent per worker $z - w(z)$ of every firm with $z > \underline{z}$ strictly decreases with κ .*
- (ii) *If $\sigma(z)$ is strictly decreasing (constant, strictly increasing) on $[\underline{z}, \bar{z}]$, the labor share is strictly increasing (constant, strictly decreasing) in κ .*

¹⁰Lindenlaub et al. (2025) obtain this formula without capital for local labor markets and use it to infer the sorting of firms across German commuting zones from local labor shares.

Proof. See Appendix A.4. □

Part (i) says that every firm raises its wage relative to its productivity when competition intensifies. Part (ii) says that the aggregate labor share need not follow. More competition moves employment toward more productive firms, and this shift of employment can lower the labor share. When $\sigma(z)$ is monotone, which of the two forces dominates depends only on the shape of the productivity distribution. Both forces are contained in the reallocation term of Proposition 1, which is the entire response under wage posting. The monotonicity of $\sigma(z)$ in part (ii) is a sufficient condition. Equation (19) with $\tau(z) = 1$ gives the exact local condition. Because $\epsilon_\ell^K(z)$ is increasing in z , a decreasing $\sigma(z)$ makes the covariance negative, but a $\sigma(z)$ that is not monotone can also do so. The distinction between the two forces corresponds to the within-producer and reallocation components of the labor share emphasized by Autor et al. (2020) and, for establishments, by Kehrig and Vincent (2021).

4.2 Sequential Auctions

Proposition 4 (Sequential auctions). *Under sequential auctions with $\beta \in [0, 1]$:*

- (i) *Part (i) of Assumption 1 holds. The match surplus $S(z)$ is the value, in excess of the value of unemployment, of a worker employed by a firm with productivity z who receives the entire surplus. It satisfies*

$$S(\underline{w}) = 0, \quad S'(z) = \frac{1}{\delta + \beta\lambda^E(1 - \hat{F}(z))}, \quad (23)$$

and the reservation level satisfies

$$\underline{w} - b = \beta(\lambda^U - \lambda^E) \int_{\underline{z}}^{\bar{z}} S(z) d\hat{F}(z). \quad (24)$$

- (ii) *The value to a firm with productivity z of hiring a worker whose competing offer comes from a firm with productivity $z' < z$ is*

$$J(z', z) = (1 - \beta)[S(z) - S(z')], \quad (25)$$

where $z' = \underline{w}$ for a worker hired from unemployment.

(iii) *The profit multiplier is*

$$\tau(z) = (1 - \beta) \frac{1 + \kappa(1 - \hat{F}(z))}{1 + \beta\kappa(1 - \hat{F}(z))}, \quad (26)$$

and the least productive active firm earns $\pi(\underline{z}) = 0$.

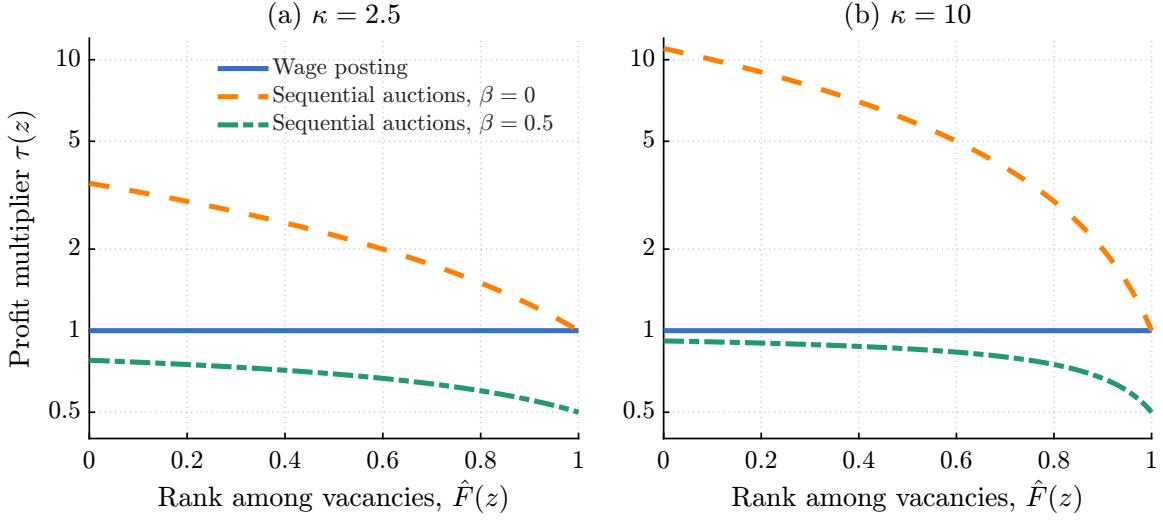
Proof. See Appendix A.5. □

Recall that $\tau(z) = \pi'(z)/\ell(z)$ is the difference in profits between firms on adjacent rungs per unit of the extra output of their workers; under wage posting, it equals one, and under sequential auctions, it can be larger or smaller. Figure 2 plots $\tau(z)$ against the rank of the firm among vacancies, $\hat{F}(z)$, for two levels of competition.

Consider first $\beta = 0$. A firm that poaches a worker must match the full surplus $S(z)$ of the current employer, and the productivity of the current employer becomes the reference value in later auctions. For the worker and the current employer together, a move is therefore not a loss. Their joint value is the same as if the worker stayed and produced z until the job is destroyed, which takes $1/\delta$ on average. One more unit of productivity raises this joint value by $1/\delta$. When the current employer hired the worker, the worker's value was set by the competing offer and did not depend on z . The whole increase therefore goes to the firm: in return for the reference value, the worker accepts a low wage at the start of the job, and starting wages can fall below b (Cahuc et al., 2006). The worker stays at the firm for only $1/T(z)$ on average, so that the difference in profits is $\tau(z) = T(z)/\delta = 1 + \kappa(1 - \hat{F}(z))$ times the extra output. With $\kappa = 2.5$, it is 3.5 times the extra output at the bottom of the ladder (panel a). At the top, no other firm can poach the worker, and $\tau(z) = 1$, as under wage posting.

When $\beta > 0$, a worker who moves receives the share β of the difference between the surpluses of the new and the old match, and the multiplier is lower for two reasons. First, the firm keeps only the share $1 - \beta$ of the surplus at the time of hiring. Second, after a move, the value of the worker depends on the surplus of the old match only with the weight $1 - \beta$, as if each move ended the old match with probability β . A unit of productivity then counts for $1/(\delta + \beta\lambda^E(1 - \hat{F}(z)))$ on average instead of $1/\delta$. Apart from the firm's share $1 - \beta$, $\tau(z)$ is therefore the length of time for which the firm's productivity counts in the joint value, relative to the length of time the worker spends at the firm. At the top of the ladder, $\tau = 1 - \beta$. With $\beta = 0.5$, τ is below one at every rank.

Figure 2: The Profit Multiplier along the Job Ladder



Notes: The profit multiplier $\tau(z)$ of Propositions 2 and 4 against the rank $\hat{F}(z)$ of the firm among vacancies, for two values of κ . The multiplier is shown on a logarithmic scale, which is the same in both panels.

Two properties of $\tau(z)$ under sequential auctions matter for the response to competition in Proposition 1. First, $\tau(z)$ falls along the ladder, because fewer firms can poach the workers of a more productive firm. Second, more competition raises $\tau(z)$ at every firm below the top, as the comparison of the two panels shows. The worker spends less time at the firm relative to the time for which the firm's productivity counts, and accepts a lower starting wage in return. By equation (26), the elasticity in equation (17) is

$$\epsilon_{\tau}^{\kappa}(z) = \frac{(1 - \beta) t}{(1 + t)(1 + \beta t)} \geq 0, \quad t \equiv \kappa(1 - \hat{F}(z)), \quad (27)$$

which lies in $[0, 1)$, so that Proposition 1 applies, and is smaller when β is larger. The wage-setting term of Proposition 1 is therefore negative under sequential auctions. I refer to it as the starting-wage effect: more competition makes the reference value that a firm gives to its workers more valuable, workers accept even lower starting wages in return, and the labor share falls.¹¹

Corollary 2 (No bargaining power). *Under sequential auctions with $\beta = 0$, the reservation*

¹¹Strictly, the effect is not limited to the wage at the start of a job. The wage that a firm sets when it matches an outside offer also takes the value of later offers into account, and τ summarizes all of these wages (Supplemental Appendix D.5).

level is $\underline{w} = b$, the average wage of employed workers equals b , and the labor share equals b divided by the average productivity of employed workers.

Proof. See Appendix A.6. □

Under $\beta = 0$, the unemployed receive none of the surplus of the matches they form, and their value equals the value of remaining unemployed forever. In the limit $r \rightarrow 0$, the corresponding flow value, b , equals the long-run average income of a worker, which pins down the average wage at b . Since $\underline{w} = b$ does not depend on κ and, at a fixed $\hat{F}$, competition raises the average productivity of employed workers, the labor share with $\beta = 0$ falls with competition for every productivity distribution.

Under sequential auctions with $r > 0$, discounting changes the profit multiplier and, through $S(z)$, the reservation level (Proposition D.1 in Supplemental Appendix D.5). The multiplier $\tau(z)$ takes the form of equation (26) with $\kappa_r \equiv \lambda^E / (r + \delta)$ in place of κ , whereas the allocation, and hence $\ell(z)$, depends on $\kappa = \lambda^E / \delta$ as before. With discounting, later auctions weigh less in the value of a job, and workers give up less of the starting wage in return for future raises. Because a higher meeting rate λ^E raises κ and κ_r in proportion, the labor share under Pareto productivity in Proposition 5 below still falls when λ^E rises, for every $r \geq 0$ (Supplemental Appendix D.5).

When the reallocation term raises the labor share, the starting-wage effect works against it. With Pareto productivity, it breaks the neutrality of wage posting.

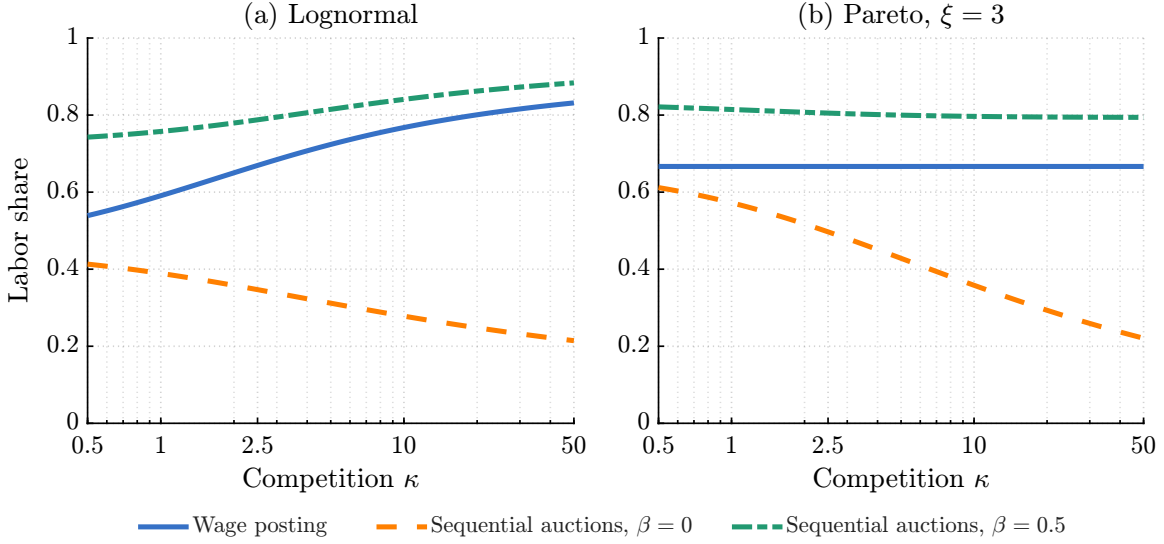
Proposition 5 (Pareto productivity under the two protocols). *Suppose that $\hat{F}(z) = 1 - (z/\underline{z})^{-\xi}$ on $[\underline{z}, \infty)$ with $\xi > 1$. Under wage posting, $LS = 1 - 1/\xi$ for all κ . Under sequential auctions with $\beta \in [0, 1)$, LS is strictly decreasing in κ , with*

$$\lim_{\kappa \rightarrow 0} LS = 1 - \frac{1 - \beta}{\xi}, \quad \lim_{\kappa \rightarrow \infty} LS = \beta^{1/\xi}. \quad (28)$$

Proof. See Appendix A.7. □

Importantly, the neutrality of Corollary 1 is a property of wage posting. In terms of Proposition 1, the reallocation term is positive only because $\tau(z)$ falls along the ladder, and the wage-setting term is larger in absolute value. With thinner tails, the reallocation term favors a higher labor share, and the sign under sequential auctions depends on the bargaining power β . Figure 3 plots the labor share against competition under the two protocols, for

Figure 3: Labor Share and Competition under Wage Posting and Sequential Auctions



Notes: The labor share from Lemma 2 with the profit multipliers of Propositions 2 and 4, with $\lambda^U = \lambda^E$, $\hat{F}$ fixed, and $b = \underline{w} = \underline{z}$. The distributions are those of Figure 1.

a lognormal and a Pareto distribution of productivity. For the lognormal distribution, the threshold β^* at which the sign changes is 0.08 at $\kappa = 2.5$ and 0.02 at $\kappa = 10$. It is higher when competition is weaker, 0.19 at $\kappa = 0.5$. For the Pareto distribution, it does not exist.

4.3 Comparison with Product Markets

The two forces of Section 4.1 have counterparts in product markets with variable markups, where tougher competition lowers the markup of every firm and shifts sales toward more productive firms, which charge higher markups (Melitz and Ottaviano, 2008; Arkolakis et al., 2019; Autor et al., 2020). The next proposition states wage posting in the language of product markets.

Proposition 6 (Wage posting as a product market). *Let $D(p) \equiv 1 - \hat{F}(p)$ denote the share of vacancies whose productivity is at least p , and let $\varphi(p) \equiv p - (1 - \hat{F}(p))/\hat{f}(p)$ denote the marginal revenue along D .*

(i) *Under wage posting,*

$$\text{LS} = \mathbb{E}^Y \left[\frac{\varphi(z)}{z} \right], \quad \frac{\varphi(z)}{z} = 1 - \frac{1}{\xi(z)} = 1 - \sigma(z). \quad (29)$$

(ii) *The tail thickness $\sigma(z)$ is decreasing if and only if the price elasticity of D increases with p , and if and only if the hazard rate of $\log z$ under $\hat{F}$ is increasing.*

Proof. See Appendix A.8. □

Part (i) reads the labor share off a demand curve, following [Bulow and Roberts \(1989\)](#). Order firms by productivity and treat $D(p)$ as the quantity demanded at price p . The local Pareto exponent $\xi(z)$ is the price elasticity of this demand curve, and the tail thickness $\sigma(z) = 1/\xi(z)$ is its inverse. Where $\sigma(z) < 1$, it equals the Lerner index of a monopolist whose optimal price is z . The labor share is therefore the output-weighted mean of the ratio of marginal revenue to price along this firm demand curve.

Part (ii) states the condition of Proposition 3(ii) in two well-known forms. The first is Marshall's second law of demand applied to the firm demand curve: as the price rises, demand becomes more elastic, and the ratio of marginal revenue to price rises with the price. The second is an increasing hazard rate of log productivity, which holds when the density of log productivity is log-concave, as when productivity is lognormal ([Bagnoli and Bergstrom, 2005](#)). In product markets, the response depends on two primitives. One is Marshall's second law, an assumption on consumer demand that makes markups rise with productivity. The other is the shape of the productivity distribution, which then determines the sign. In the model of [Autor et al. \(2020\)](#), tougher competition raises the aggregate labor share if the density of log productivity is log-concave. Under wage posting, the demand curve that a firm faces is generated by the distribution of productivity across vacancies. By part (ii), Marshall's second law for this curve is then itself a condition on that distribution, an increasing hazard rate of log productivity, which log-concavity implies. With Pareto productivity, the two markets have the same neutrality: the distribution of markups does not depend on trade costs ([Arkolakis et al., 2019](#)), and Corollary 1 is the counterpart for the labor share.

What distinguishes labor markets from these product-market models is the wage-setting protocol. Under sequential auctions, more competition lowers the starting wages that workers accept in return for future raises. This effect comes from the path of wages within a job and has no counterpart in static product-market models. It breaks the neutrality under Pareto productivity (Proposition 5).

5 Measurement

Lemma 2 and Proposition 1 express the aggregate labor share and its response to competition through the vacancy-weighted distribution $\hat{F}$, which is not observed. In this section, I first add capital. I then express the aggregate labor share through two objects: the distribution of productivity across employed workers, which establishment-level data measure, and the degree of competition. Next, I read the productivity distributions of U.S. manufacturing and retail trade through these results. I end this section by quantifying the impact of the decline in outside offers to employed workers since the 1980s. Supplemental Appendices B and C describe the data and the computations.

5.1 Capital

The model has no capital, whereas in the data, part of value added pays for capital. Nevertheless, capital can be added without changing the analysis if firms rent it in a competitive market and adjust it freely. Suppose that a worker employed by a firm with total factor productivity a produces value added $y = a k^\alpha$ with capital k , which the firm rents at the user cost r_K , the interest rate plus the depreciation rate. Given its workers, the firm chooses $k = (\alpha a / r_K)^{1/(1-\alpha)}$, and its net output per worker z , value added net of capital costs, is

$$z \equiv \max_k \{a k^\alpha - r_K k\} = (1 - \alpha) \left(\frac{\alpha}{r_K} \right)^{\alpha/(1-\alpha)} a^{1/(1-\alpha)} = (1 - \alpha) y. \quad (30)$$

Net output is linear in employment, and the profit of a firm per worker is $z - w$. Under both protocols, the labor market is therefore the one of the model with z as productivity, and all the results apply to it. Payments to capital are α times value added, and the labor share of value added is $(1 - \alpha)$ LS, where LS is the labor share of net output. Value added per worker is $z/(1 - \alpha)$, and the distribution of value added per worker across workers is $L(z)$ up to scale, which does not affect LS. Because α does not depend on κ , a change in competition changes the labor share of value added by $1 - \alpha$ times the change in LS.

Because $r_K k = \alpha y$, α is r_K times the ratio of the capital stock to value added. [Gouin-Bonenfant \(2022\)](#) sets α in this way with Canadian firm data, and I measure the ratio in U.S. manufacturing. In the accounts of the Bureau of Economic Analysis ([U.S. Bureau of Economic Analysis, 2025, 2026](#)), the current-cost net stock of private fixed assets in manufacturing averages 1.80 times value added in 2013–2019, and its depreciation is 11.6

percent of the stock. With an interest rate of 0.05, $r_K = 0.05 + 0.116$ and $\alpha = 0.166 \times 1.80 = 0.30$. Inventories and land, which the stock excludes, would raise α . I set $\alpha = 0.3$, and the labor shares that I compute from the data are shares of value added.

5.2 The Distribution of Employment

Let $\mu_L \equiv \int z dL(z)$ denote output per employed worker. The following proposition expresses the labor share in terms of what can be measured from the data for each wage-setting protocol.

Proposition 7 (Labor share and the distribution of employment). *Under wage posting,*

$$1 - \text{LS} = \frac{1}{\mu_L} \int_{\underline{z}}^{\bar{z}} (1 - L(z)) \frac{1 + \kappa L(z)}{1 + \kappa} dz. \quad (31)$$

Under sequential auctions with $\beta \in [0, 1)$,

$$1 - \text{LS} = \frac{1 - \beta}{\mu_L} \int_{\underline{z}}^{\bar{z}} \frac{(1 - L(z))(1 + \kappa L(z))}{1 + \kappa L(z) + \beta \kappa (1 - L(z))} dz. \quad (32)$$

Proof. See Appendix A.9. □

The proof substitutes equation (5) into Lemma 2. Proposition 7 replaces the unobserved $\hat{F}$ with two observable objects. The first is the distribution of productivity across workers $L(z)$, which is the distribution of establishment-level productivity weighted by employment or hours. The second is the degree of competition κ , which can be measured from worker flows. Given the wage-setting protocol, these two objects determine the labor share. Because equation (5) recovers $\hat{F}$ from them, they also determine the objects in Proposition 1 that govern its response to competition at a fixed $\hat{F}$.

5.3 The Shape of Productivity Distributions

Data. I use the Dispersion Statistics on Productivity (DiSP) of the Bureau of Labor Statistics and the Census Bureau. For each of the 86 four-digit NAICS manufacturing industries and each year from 1987 to 2021, the data provide differences between percentiles of the within-industry distribution of log productivity across establishments ([U.S. Bureau](#)

of Labor Statistics and U.S. Census Bureau, 2025; Cunningham et al., 2023). I use real gross output per hour, or labor productivity, with the statistics weighted by hours, which describe the distribution of productivity across hours worked. For retail trade, the DiSP reports finer percentile steps every five years from 1987 to 2017, which show the intervals that the manufacturing data do not separate. The DiSP also reports total factor productivity (TFP), which I discuss at the end of Section 5.4.

From the DiSP to $L(z)$. Consider the establishments j of an industry in a year, with real gross output Q_j , value added VA_j in the same prices, hours H_j , and N_j workers, and let $x_j \equiv \log(Q_j/H_j)$. The DiSP reports percentile gaps $x_{p_2} - x_{p_1}$ of the hours-weighted distribution

$$L^H(x) \equiv \frac{\sum_j H_j \mathbb{1}\{x_j \leq x\}}{\sum_j H_j}, \quad L^H(x_p) = p. \quad (33)$$

In the model, establishment j is a firm whose productivity z_j is its net output per worker $(1 - \alpha)VA_j/N_j$, and it employs $N_j = v_j \ell(z_j)$ workers, so that $L(z) = \sum_j N_j \mathbb{1}\{z_j \leq z\} / \sum_j N_j$.

The two distributions, $L(z)$ and $L^H(x)$, describe the same object under four conditions. First, establishments are the rungs of the ladder: productivity is a property of the establishment, and workers move between establishments as in Section 2. Second, hours per worker, H_j/N_j , are the same for all establishments of an industry.¹² Third, the share of value added in gross output, VA_j/Q_j , is also the same for all establishments of an industry.¹³ Fourth, each industry is a labor market in a steady state with the degree of competition κ , and the DiSP measures the permanent productivity of the model.

To see that, note from Section 5.1 that $z_j = (1 - \alpha)y_j$, where y_j is establishment j 's

¹²The condition concerns the average hours of an establishment's workers, not individual hours. In administrative data from Washington State, Lachowska et al. (2026) find that employer effects account for about a quarter of the variance of log hours. Within sectors, employer effects on hours are almost uncorrelated with employer effects on hourly wages, with a correlation of 0.05. Hours per worker therefore do not rise systematically with the pay of the employer.

¹³This condition holds if intermediate inputs enter like capital in Section 5.1. Suppose that a worker produces gross output $a k^{\alpha_K} m^{\alpha_M}$ with intermediate inputs m that the firm buys at a common price and adjusts freely. Then value added is the fraction $1 - \alpha_M$ of gross output at every firm. Net output is $1 - \alpha$ times value added with $\alpha = \alpha_K / (1 - \alpha_M)$, and α remains the share of capital costs in value added.

value added per worker VA_j/N_j . I can write z_j as

$$z_j = (1 - \alpha) \frac{VA_j}{N_j} = (1 - \alpha) \times \frac{H_j}{N_j} \times \frac{VA_j}{Q_j} \times \underbrace{\frac{Q_j}{H_j}}_{\exp(x_j) \text{ in DiSP}}.$$

The first factor is the same for all firms, and the second and third conditions make the second and third factors common to all establishments of an industry. Because hours per worker are common, the hours weights in equation (33) are also employment weights, and the first and fourth conditions then give

$$L^H(x) = L(\chi \cdot \exp(x)), \quad (34)$$

where χ is a constant. By equation (34), the percentile gaps of the DiSP are those of log productivity under L . The constant χ does not matter because Lemma 2 and Proposition 1 do not depend on the scale of productivity, and I can use equation (5) to recover $\hat{F}$ from L^H and κ :

$$\hat{F}(\chi \cdot \exp(x)) = \frac{(1 + \kappa) L^H(x)}{1 + \kappa L^H(x)}. \quad (35)$$

Tail thickness in data. Let $\sigma_L(z) \equiv (1 - L(z))/(z L'(z))$ denote the tail thickness of $L(z)$ at z , which is the slope of log productivity against $-\log(1 - L(z))$. Its average between the percentiles $p_1 < p_2$ of $L(z)$ is

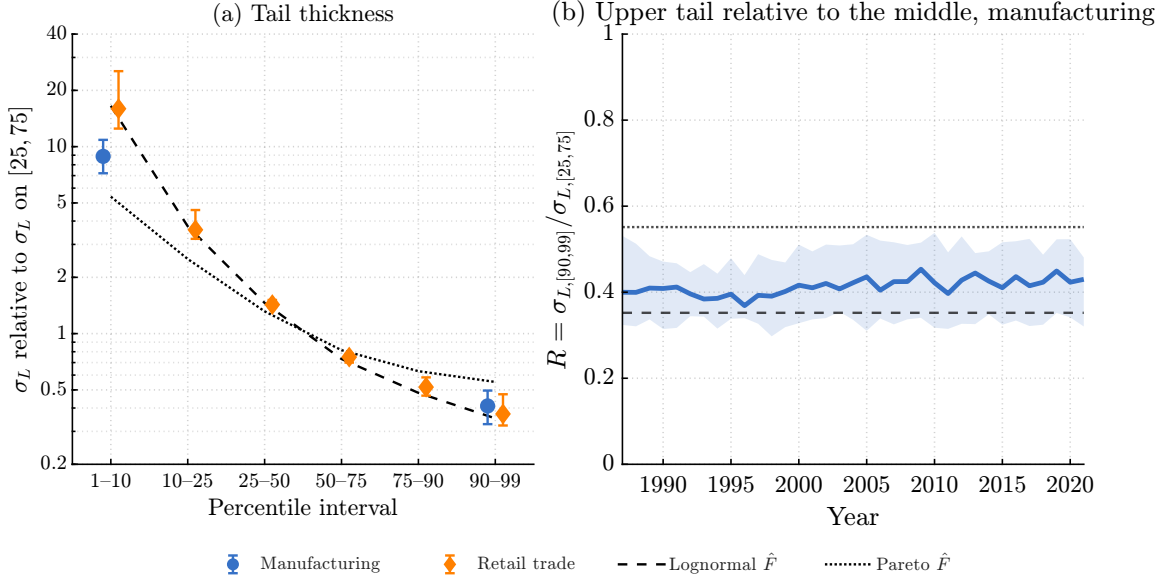
$$\sigma_{L,[p_1,p_2]} = \frac{x_{p_2} - x_{p_1}}{\log((1 - p_1)/(1 - p_2))}, \quad (36)$$

where x_p denotes the p -th percentile of log productivity. In subscripts, I write p in percent, so that $\sigma_{L,[90,99]}$ is the average between $p_1 = 0.9$ and $p_2 = 0.99$. For each industry and year, I compute the ratio

$$R \equiv \frac{\sigma_{L,[90,99]}}{\sigma_{L,[25,75]}}, \quad (37)$$

which equals one for a Pareto distribution and 0.37 for a lognormal distribution, whatever the dispersion, and does not change when productivity is raised to a power. The theory, however, concerns the shape of $\hat{F}$, and by equation (5), the tail thicknesses of the two

Figure 4: The Tail Thickness of Productivity in U.S. Manufacturing and Retail Trade



Notes: DiSP, output per hour weighted by hours, manufacturing 1987–2021 and retail trade 1987–2017 (every five years). Panel (a): the tail thickness σ_L on each percentile interval, from equation (36), relative to its value on [25, 75]. Markers show medians and bars interquartile ranges across industry-years, weighted by industry employment in the Business Dynamics Statistics (BDS) of the Census Bureau (U.S. Census Bureau, 2025) for manufacturing and by the industry share of hours for retail trade. The DiSP reports the intervals [1, 10], [25, 75], and [90, 99] for manufacturing and all six intervals for retail trade. Lines in both panels: the benchmarks of a lognormal and a Pareto $\hat{F}$ at $\kappa = 18.4$. Panel (b): median and interquartile range across industries of R in each year, weighted by industry employment in the BDS. Supplemental Appendix B.1 describes the data.

distributions are related by

$$\sigma(z) = \sigma_L(z) \frac{1 + \kappa L(z)}{1 + \kappa}. \quad (38)$$

The factor rises from $1/(1 + \kappa)$ at the bottom of the ladder to one at the top, and σ therefore declines less steeply than σ_L . The reason is that workers climb the ladder. The more often they receive outside offers, the faster they leave unproductive firms and the more they accumulate at productive ones. Low on the ladder, few workers remain relative to those employed above, and L has a thicker tail there than $\hat{F}$, whereas near the top the two distributions have the same shape. At $\kappa = 18.4$, the value implied by U.S. worker flows (Section 5.4), a Pareto $\hat{F}$ implies $R = 0.55$ and a lognormal $\hat{F}$ implies $R = 0.35$, again whatever the dispersion.¹⁴ Figure 4 uses these two cases as benchmarks.

¹⁴The values for the lognormal distribution, here and above, are those without truncation. Truncating the distribution from below changes them.

Panel (a) of Figure 4 shows that $\sigma_L(z)$ falls from the bottom to the top of the productivity distribution more steeply than a Pareto $\hat{F}$ implies. For labor productivity in manufacturing, the median of R across industry-years is 0.41, with an interquartile range from 0.33 to 0.50, between the two benchmarks and closer to the lognormal one. The local Pareto exponent of $L(z)$ on $[90, 99]$, $1/\sigma_{L,[90,99]}$, has a median of about 3.6. In retail trade, where all six steps between the percentiles 1, 10, 25, 50, 75, 90, and 99 are observed, $\sigma_L(z)$ falls monotonically and follows the profile of a lognormal $\hat{F}$ closely in every interval.

Panel (b) shows the median and interquartile range across manufacturing industries of R over the years. The ratio has changed little since 1987.

5.4 The Response in U.S. Manufacturing

To evaluate Proposition 1 in the data, I construct a continuous distribution $L(z)$ for each industry and each year from 2013 to 2019. The percentiles are joined by a monotone interpolation of log productivity in $-\log(1-p)$. Because the DiSP does not report where the interquartile range lies between x_{10} and x_{90} , I divide the remaining gap $(x_{90}-x_{10})-(x_{75}-x_{25})$ equally between $[10, 25]$ and $[75, 90]$. The distribution is extended above the 99th percentile by a Pareto tail with the exponent $1/\sigma_{L,[90,99]}$.¹⁵ The lowest productivity is the first percentile, and $\underline{w} = \underline{z}$. I set $\kappa = 18.4$, the value implied by worker flows in the Current Population Survey (CPS) for the same years.¹⁶ I weight industry-years by their value added in the Annual Survey of Manufactures (ASM) of the Census Bureau (U.S. Census Bureau, 2026), so that averages are those of manufacturing as a whole (Supplemental Appendix B.1). I recover $\hat{F}$ from L^H with equation (35), and compute the labor share of value added and its derivative with respect to $\log \kappa$ at a fixed $\hat{F}$.

Results. Table 1 compares the labor share of value added that the model implies, without targeting it, with that in the data. The data lie between the levels under wage posting and under sequential auctions with $\beta = 0.5$.

¹⁵The DiSP reports no percentile above the 99th. I assume that log productivity keeps rising linearly in $-\log(1-p)$ above the 99th percentile, with the slope of the interval $[90, 99]$. The integrals that define output and profits are computed numerically far into this tail, and the remainder is added analytically (Supplemental Appendix B.2).

¹⁶It is the value at which the stationary job ladder reproduces the ratio of the job-to-job rate to the separation rate into unemployment of private wage and salary workers (Supplemental Appendix C.1).

Table 1: The Labor Share of Value Added in U.S. Manufacturing, 2013–2019

Data	Model		
	Posting	Sequential auctions	
		$\beta = 0.5$	$\beta = 0$
0.49	0.42	0.53	0.12

Notes: Data: compensation of employees relative to value added in the GDP-by-industry accounts of the Bureau of Economic Analysis ([U.S. Bureau of Economic Analysis, 2026](#)). Model: DiSP manufacturing industry-years, hours-weighted distributions of output per hour, $\kappa = 18.4$; averages weighted by industry value added in the ASM.

Table 2: Labor Share and Competition Implied by the DiSP Distributions

Derivative with respect to $\log \kappa$						Threshold β^*
Posting		Sequential auctions				
Median	Share > 0	$\beta = 0.5$		$\beta = 0$		
		Median	Share > 0	Median	Share > 0	Median
0.020	0.97	0.011	0.97	-0.041	0.00	0.07

Notes: DiSP, manufacturing industry-years, 2013–2019, with productivity measured by output per hour. Medians and shares across industry-years, weighted by industry value added in the ASM. The threshold β^* is searched on $[0, 0.95]$, and industry-years without a threshold in this range count as above 0.95.

Table 2 reports the derivatives. Under wage posting, more competition raises the labor share in 97 percent of industry-years. At the median, a 10 log-point increase in κ raises the labor share of value added by about 0.20 percentage points.

Under sequential auctions, the sign depends on bargaining power. More competition lowers the labor share in every industry-year when $\beta = 0$, as Corollary 2 implies, and raises it in almost all of them when $\beta = 0.5$. The threshold β^* at which the sign changes is small, with a median of 0.07. I report $\beta = 0.5$, equal bargaining power, as a benchmark; β^* gives the sign for almost any other value.

In the literature, some estimates of bargaining power are close to zero. [Cahuc et al. \(2006\)](#) estimate a model of sequential auctions with bargaining on French matched employer-employee data for 1993–2000. Depending on the industry, they find bargaining power between 0 and 0.2 for workers without managerial tasks and between 0.2 and 0.4 for skilled workers (supervisors and engineers). With Danish data and a model that adds human capital accumulation, [Bagger et al. \(2014\)](#) estimate about 0.3 for all three education groups. For

workers with little bargaining power, the wage-setting protocol can therefore determine the sign of the response. For manufacturing as a whole, however, bargaining power close to zero is hard to reconcile with the level of the labor share. With $\beta = 0$, the labor share of value added is far below that in the data, as shown in Table 1.¹⁷

TFP-based measure. In the DiSP, TFP is measured as real gross output per unit of an index of capital, labor, and intermediate inputs weighted by their shares in costs. It corresponds to the productivity of the model only under further conditions on capital and intermediate inputs per hour, which Supplemental Appendix B.3 discusses. The upper tail of TFP is thicker than that of output per hour: in 2013–2019, the median of R is 0.60 for TFP and 0.39 for output per hour. The results are therefore weaker. Depending on how the dispersion of TFP is scaled, more competition raises the labor share under wage posting in 57 to 61 percent of industry-years. Under sequential auctions, it lowers the labor share in 37 to 41 percent of industry-years even with $\beta = 0.95$.

5.5 The Decline in Outside Offers

Section 5.4 evaluates the response to a small change in competition. This subsection evaluates a large one, the decline in outside offers to employed workers since the 1980s. Engbom et al. (2026) estimate that the rate at which employed workers receive better-paying outside offers fell by about half between the 1980s and the 2010s, and they relate the decline to employer concentration and noncompete agreements. I compare the economy of 2013–2019 with an economy in which employed workers receive outside offers at the rate of the 1980s, about twice the current rate. The comparison isolates the decline in outside offers and is not an estimate of the historical transition.

In Table 3, the job-to-job rate falls by less than half as much as the offer rate, because workers who receive fewer offers are lower on the ladder and accept a larger share of the offers they receive. On average, the decline in offers lowers the labor share by 1.8 percentage points under wage posting and by 0.9 points under sequential auctions with $\beta = 0.5$. Under both protocols, it lowers the labor share in 96 percent of industry-years, weighted by value added. With $\beta = 0$, it raises the labor share in every industry-year, as Corollary 2 implies.

¹⁷Across U.S. states, Engbom et al. (2026) also find higher wages and a higher labor share where κ is higher. With $\beta = 0$, in contrast, the average wage equals b at any level of competition, and the labor share falls with competition (Corollary 2).

Table 3: The Decline in Outside Offers and the Labor Share

	JJ rate	Competition κ	Labor share		
			Posting	Sequential auctions	
				$\beta = 0.5$	$\beta = 0$
2013–2019	2.30	18.4	0.42	0.53	0.12
Decline in offers since the 1980s	–22%	–51%	–1.8 p.p.	–0.9 p.p.	+2.5 p.p.

Notes: DiSP manufacturing industry-years of 2013–2019, hours-weighted distributions of output per hour; averages weighted by industry value added in the ASM. The first row reports the monthly job-to-job (JJ) rate of private wage and salary workers in percent (CPS), competition κ , and the labor share of value added. The second row reports the change from an economy with the offer rate of the 1980s.

Supplemental Appendix C.2 repeats the comparison with alternative assumptions on separations, on the level of competition, on the persistence of the measured dispersion, and on the division of the percentile gaps, and with a decline of a third. Under wage posting, the decline in offers lowers the labor share in all of them, by between 1.0 and 5.1 points, and under sequential auctions with $\beta = 0$, it raises the labor share in all of them.

Two caveats apply. First, Sections 5.4 and 5.5 combine the productivity distributions of manufacturing with the flows of private wage and salary workers in all industries. Second, Engbom et al. (2026) infer the decline in offers from the gap between the wages of all workers and those of workers hired from nonemployment, using the steady state of a job ladder in which each job pays a fixed wage. Their extension with wage growth within jobs gives a similar decline. It does not include raises triggered by outside offers, as under sequential auctions, and the same data could imply a different decline.

6 Vacancy Creation

So far, the vacancy-weighted distribution has been fixed. This section lets firms choose vacancies. The accounting results hold for any allocation and any vacancy-weighted distribution, and they therefore continue to hold. Given $\hat{F}$ and the reservation level, the labor share depends on the meeting rates only through κ , which Section 5 takes from the data. The question is therefore how the reallocation of vacancies across firms changes the labor share.

I assume that a firm with productivity z posts v vacancies at cost $c(v) = \bar{c}v^{1+\omega}/(1+\omega)$,

with $\omega > 0$. In the limit $r \rightarrow 0$, the firm chooses $v(z)$ to maximize $v \pi(z) - c(v)$, which gives $c'(v(z)) = \pi(z)$.

Endogenous responses of vacancies to competition change the vacancy-weighted distribution $\hat{F}$ and hence σ . Because vacancies rise with profits, which rise with productivity, $\hat{F}$ is not Pareto even when the productivity distribution across firms Ψ is, and a change in competition reshapes $\hat{F}$. With Pareto productivity and wage posting, however, this reallocation leaves the labor share unchanged.

Proposition 8 (Pareto productivity with vacancy creation under wage posting). *Suppose that firms post wages and that Ψ is Pareto, $\Psi(z) = 1 - (z/z_0)^{-\xi}$ on $[z_0, \infty)$ with $\xi > 1 + 1/\omega$. Then, in every steady state,*

$$\text{LS} = 1 - \frac{1 + 1/\omega}{\xi}, \quad (39)$$

whatever the meeting and separation rates.

Proof. See Appendix A.10. □

Under wage posting, profits per vacancy rise with productivity at the rate of employment per vacancy, $\pi'(z) = \ell(z)$, and the vacancy condition ties profits to vacancies, $\pi(z) = c'(v(z))$. With a Pareto density, these two conditions make aggregate output proportional to aggregate profits, whatever the allocation. As $\omega \rightarrow \infty$, vacancies no longer respond to profits, $\hat{F}$ is Ψ truncated at $\underline{z}$, which is Pareto with the same exponent, and equation (39) reduces to Corollary 1. With a finite ω , more productive firms post more vacancies, which thickens the upper tail of $\hat{F}$ and lowers the labor share. The fall equals the vacancy costs: profits net of these costs remain $1/\xi$ of output, as in Corollary 1. Changes in competition then leave the labor share unchanged.

Outside Pareto productivity and wage posting, the reallocation of vacancies changes the labor share and its response to competition to the extent that vacancies respond to profits. Sections 3 to 5 hold $\hat{F}$ fixed, which is the limit $\omega \rightarrow \infty$: vacancies do not respond to profits, and $\hat{F}$ does not change with competition.¹⁸ Some existing evidence points to a weak response, so that holding $\hat{F}$ fixed is a good approximation. Engbom (2023) estimates a curvature of the vacancy cost of about 18 from the relatively low share of vacancies posted

¹⁸The vacancy-weighted distribution is then a primitive, as in Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002) and in the empirical models that recover it from data (Bontemps et al., 2000; Cahuc et al., 2006).

by the most productive Swedish firms, and makes the same assumption in his analytical results. [Bagger and Lentz \(2019\)](#) estimate that the elasticity of recruiting intensity with respect to the value of recruiting is 0.006 in Danish matched data, so that the vacancy-weighted distribution $\hat{F}$ is close to the distribution of productivity across firms Ψ . In [Gouin-Bonenfant \(2022\)](#), endogenous vacancy rates calibrated to the establishment-level evidence of [Davis et al. \(2013\)](#) leave the response of the labor share to productivity dispersion almost unchanged. In Supplemental Appendix [B.4](#), with $\kappa = 18.4$ and under wage posting, the labor share of value added with vacancy creation matched to the employment share of large establishments is within 2 percentage points of that with fixed vacancies. Estimates from aggregate fluctuations and from the size distribution of firms imply more elastic vacancy creation ([Lise and Robin, 2017](#); [Engbom and Moser, 2022](#)). Proposition 8 shows how a finite ω changes the labor share with Pareto productivity.

7 Conclusion

This paper studies when competition for workers raises the aggregate labor share. Competition can reduce the wage markdown at every firm, but it also moves workers up the job ladder to more productive firms, which tend to have lower labor shares. I derive an accounting identity for the labor share in which, for a given allocation of workers, the wage-setting protocol enters only through the profit multiplier. Under wage posting, competition raises the labor share if the tail thickness falls with productivity and leaves it unchanged under Pareto productivity. Under sequential auctions, workers accept lower starting wages in return for more frequent raises, and with Pareto productivity, competition lowers the labor share for any bargaining power below one.

In the DiSP distributions of labor productivity in U.S. manufacturing, the tail thickness falls with productivity. More competition then raises the labor share in almost all industry-years, as the common view holds, unless wages are set by sequential auctions with bargaining power close to zero. For a given distribution of vacancies and with the degree of competition implied by U.S. worker flows, the decline in outside offers to employed workers that [Engbom et al. \(2026\)](#) estimate lowers the labor share of manufacturing by 1.8 percentage points under wage posting. Under sequential auctions with equal bargaining power, higher starting wages dampen the effect, and the labor share falls by 0.9 points. When workers have no bargaining

power, higher starting wages more than offset the reallocation, and the decline in offers raises the labor share.

More generally, the effect of labor market competition on the labor share cannot be read off its effect on the markdown at each firm. It depends on the distribution of productivity across the firms among which competition moves workers and on how wages are set, including how they evolve within a job. Both are needed to assess any change that makes outside offers to employed workers more or less frequent, whether it comes from matching efficiency or from labor market institutions such as the enforcement of noncompete agreements.

A Proofs

A.1 Proof of Lemma 2

Since π is absolutely continuous, $\pi(z) = \pi(\underline{z}) + \int_{\underline{z}}^z \pi'(x) dx$. Using $\hat{F}(\underline{z}) = 0$ and exchanging the order of integration, which Fubini's theorem allows under the integrability conditions of Section 3.1 (Tonelli's theorem suffices when π is monotone), $\int_{\underline{z}}^{\bar{z}} \pi(z) d\hat{F}(z) = \pi(\underline{z}) + \int_{\underline{z}}^{\bar{z}} \int_{\underline{z}}^z \pi'(x) dx d\hat{F}(z) = \pi(\underline{z}) + \int_{\underline{z}}^{\bar{z}} \pi'(x)(1 - \hat{F}(x)) dx$. Dividing by $Y/V = \int_{\underline{z}}^{\bar{z}} z \ell(z) d\hat{F}(z)$ and using equation (7) gives

$$1 - \text{LS} = \frac{\pi(\underline{z}) + \int_{\underline{z}}^{\bar{z}} \pi'(x)(1 - \hat{F}(x)) dx}{\int_{\underline{z}}^{\bar{z}} z \ell(z) d\hat{F}(z)}.$$

By the definitions of τ and σ , $\pi'(x)(1 - \hat{F}(x)) = \tau(x) \sigma(x) x \ell(x) \hat{f}(x)$. Splitting the fraction, the term with the integral is therefore $\mathbb{E}^Y[\tau(z) \sigma(z)]$. Under Assumption 1(ii), $\pi(\underline{z}) = 0$, and this gives equation (15); with $\pi(\underline{z}) > 0$, it gives the expression in footnote 7. $\square$

A.2 Proof of Proposition 1

By equations (7), (10), and (11),

$$1 - \text{LS} = \frac{N}{\mathcal{D}}, \quad N \equiv \int_{\underline{z}}^{\bar{z}} f^R(z) dz, \quad \mathcal{D} \equiv \int_{\underline{z}}^{\bar{z}} f^Y(z) dz,$$

where $\mathcal{N}$ and $\mathcal{D}$ are aggregate profits and output per vacancy. Holding $\hat{F}$, $\underline{z}$, and $\underline{w}$ fixed, competition changes $f^R(z) = \tau(z)\ell(z)(1 - \hat{F}(z))$ and $f^Y(z) = z\ell(z)\hat{f}(z)$ only through $\tau(z)$ and $\ell(z)$. By equation (4),

$$\ell(z) = \ell(\underline{z}) \left(\frac{1 + \kappa}{1 + t} \right)^2, \quad t \equiv \kappa(1 - \hat{F}(z)),$$

where $\ell(\underline{z}) = (\lambda^F/\delta)(1 + \kappa)^{-2}$ does not depend on z . It therefore cancels from the ratio $\mathcal{N}/\mathcal{D}$, and $1 - \text{LS} = \tilde{\mathcal{N}}/\tilde{\mathcal{D}}$ with $\tilde{\mathcal{N}} \equiv \mathcal{N}/\ell(\underline{z})$ and $\tilde{\mathcal{D}} \equiv \mathcal{D}/\ell(\underline{z})$, whose integrands depend on the meeting and separation rates only through κ . The elasticity of $\ell(z)/\ell(\underline{z})$ with respect to κ is $2\kappa/(1 + \kappa) - 2t/(1 + t) = 2\kappa\hat{F}(z)/((1 + \kappa)(1 + t))$, which is $\epsilon_\ell^\kappa(z)$ in equation (16) and lies in $[0, 2)$. With $|\epsilon_\tau^\kappa(z)| \leq M$, the logarithms of the integrands of $\tilde{\mathcal{N}}$ and $\tilde{\mathcal{D}}$ change by at most $(2 + M) \log 2$ between κ and any κ' in the neighborhood with $\kappa/2 \leq \kappa' \leq 2\kappa$, and the integrands at κ' are therefore at most 2^{2+M} times those at κ . Their derivatives with respect to $\log \kappa'$ are bounded in absolute value by $(2 + M) 2^{2+M}$ times the integrands at κ , which are integrable, and dominated convergence allows differentiating under the integral sign:

$$\frac{d \log \tilde{\mathcal{N}}}{d \log \kappa} = \mathbb{E}^R[\epsilon_\ell^\kappa(z) + \epsilon_\tau^\kappa(z)], \quad \frac{d \log \tilde{\mathcal{D}}}{d \log \kappa} = \mathbb{E}^Y[\epsilon_\ell^\kappa(z)].$$

Hence

$$\begin{aligned} \frac{d \text{LS}}{d \log \kappa} &= -(1 - \text{LS}) \left(\frac{d \log \tilde{\mathcal{N}}}{d \log \kappa} - \frac{d \log \tilde{\mathcal{D}}}{d \log \kappa} \right) \\ &= (1 - \text{LS}) \left\{ \mathbb{E}^Y[\epsilon_\ell^\kappa(z)] - \mathbb{E}^R[\epsilon_\ell^\kappa(z)] - \mathbb{E}^R[\epsilon_\tau^\kappa(z)] \right\}. \end{aligned}$$

This is the first line of equation (18). Substituting $\epsilon_\ell^\kappa(z) = 2\kappa L(z)/(1 + \kappa)$ from equation (16) gives the second line. $\square$

A.3 Proof of Proposition 2

Let F denote the distribution of posted wages across vacancies. A vacancy that posts a wage $w \geq \underline{w}$ hires unemployed workers and employed workers who earn less than w . Repeating

the argument of Lemma 1 with wages in place of productivities, such a vacancy employs

$$\tilde{\ell}(w) = \frac{\lambda^F \delta}{(\delta + \lambda^E (1 - F(w)))^2}$$

workers at every point of continuity of F . The profit flow per vacancy of a firm with productivity z is $\pi(z) = \max_{w \geq \underline{w}} (z - w) \tilde{\ell}(w)$.

Part (i) and equation (20) are properties of the equilibrium of [Burdett and Mortensen \(1998\)](#) with heterogeneous firms ([Bontemps et al., 2000](#)), in the limit $r \rightarrow 0$. Supplemental Appendix D.3 proves them.

Part (ii). By the envelope theorem ([Milgrom and Segal, 2002](#), Theorem 2), π is absolutely continuous with $\pi'(z) = \tilde{\ell}(w(z)) = \ell(z)$ for almost every z . Since ℓ is continuous, π is continuously differentiable with $\pi' = \ell$ everywhere. This gives $\tau(z) = 1$. The least productive active firm pays $w(\underline{z}) = \underline{w} = \underline{z}$ and earns $\pi(\underline{z}) = 0$. Since $\pi(z) = (z - w(z)) \ell(z)$, integrating π' from $\underline{z}$ to z gives equation (21). Conversely, the wage in equation (21) is optimal: posting $w(z')$ gives a firm with productivity z the profit $\pi(z') + (z - z') \ell(z')$, and $\pi(z) - \pi(z') - (z - z') \ell(z') = \int_{z'}^z (\ell(x) - \ell(z')) dx \geq 0$ because ℓ is increasing.

Part (iii) follows from Lemma 2. □

A.4 Proof of Proposition 3

Part (i). By equation (21), $z - w(z) = \int_{\underline{z}}^z \ell(x) / \ell(z) dx$, where

$$\frac{\ell(x)}{\ell(z)} = \left[\frac{1 + \kappa(1 - \hat{F}(z))}{1 + \kappa(1 - \hat{F}(x))} \right]^2.$$

For $q_1 < q_2$, $\partial[(1 + \kappa q_1)/(1 + \kappa q_2)] / \partial \kappa = (q_1 - q_2) / (1 + \kappa q_2)^2 < 0$. Hence $\ell(x) / \ell(z)$ strictly decreases with κ whenever $\hat{F}(x) < \hat{F}(z)$, and $z - w(z)$ strictly decreases for $z > \underline{z}$.

Part (ii). By equation (22), $1 - \text{LS} = \mathbb{E}^Y[\sigma(z)]$. For $\kappa' > \kappa$, the ratio of the output weights is proportional to $[(1 + \kappa q)/(1 + \kappa' q)]^2$ with $q = 1 - \hat{F}(z)$, which is strictly decreasing in q and hence strictly increasing in z . The output weights under κ' therefore dominate those under κ in the monotone likelihood ratio order, and hence in the first-order stochastic order, and the two distributions differ. The mean of a strictly decreasing (constant, strictly increasing) σ therefore strictly decreases (stays constant, strictly increases). □

A.5 Proof of Proposition 4

Parts (i) and (ii) are the surplus-sharing rule of Cahuc et al. (2006), which reduces to that of Postel-Vinay and Robin (2002) when $\beta = 0$, in the limit $r \rightarrow 0$. Supplemental Appendix D.4 derives them. Part (iii) uses two of their properties. First, the joint value of the worker and the current employer, in excess of the value of unemployment, does not depend on how they divide it and equals $S(z)$. Second, $J(z', z)$ in equation (25) therefore already accounts for the later renegotiations and for the loss of the worker to a more productive firm.

Part (iii). Each hire generates a flow of profits until the worker leaves the firm. In a steady state, hires arrive at a constant rate, and the current profit flow of a vacancy adds up the flows of all its past hires who are still employed. It therefore equals the rate of hires times the expected total profits of one hire, which equal $J(z', z)$ in the limit $r \rightarrow 0$. The competing offer of a hire of firm z is distributed according to $\hat{G}$ on $[\underline{w}, z)$: it has a mass $\hat{G}(\underline{z}) = \delta/(\delta + \lambda^E)$ at $\underline{w}$, corresponding to unemployed workers, and a density on $(\underline{z}, z)$. Hence

$$\pi(z) = \lambda^F \int_{[\underline{w}, z)} J(z', z) d\hat{G}(z') = \lambda^F (1 - \beta) \left[S(z) \hat{G}(z) - \int_{[\underline{w}, z)} S(z') d\hat{G}(z') \right].$$

Differentiating gives $\pi'(z) = \lambda^F (1 - \beta) S'(z) \hat{G}(z) = (1 - \beta) S'(z) h(z)$. Dividing by $\ell(z) = h(z)/T(z)$ gives $\tau(z) = (1 - \beta) T(z)/(\delta + \beta \lambda^E (1 - \hat{F}(z)))$, which is equation (26). At $z = \underline{z} = \underline{w}$, $S(\underline{z}) = 0$, and hence $\pi(\underline{z}) = 0$. $\square$

A.6 Proof of Corollary 2

With $\beta = 0$, equation (24) gives $\underline{w} = b$, and hence $\underline{z} = b$, and equation (26) gives $\tau(z)\ell(z) = \lambda^F/T(z)$. By the proof of Lemma 2, profits per unit of $V\lambda^F$ are

$$\int_{\underline{z}}^{\bar{z}} \frac{1 - \hat{F}(z)}{T(z)} dz,$$

and output net of b times employment, per unit of $V\lambda^F$, is $\int_{\underline{z}}^{\bar{z}} (z - b) \delta \hat{f}(z)/T(z)^2 dz$. Both integrals are finite when output is finite, because $T \geq \delta$. The two integrands differ by the derivative of a function that vanishes at both ends of the distribution. Let $H(z) \equiv$

$-(z-b)(1-\hat{F}(z))/T(z)$. Since $d[(1-\hat{F}(z))/T(z)]/dz = -\delta\hat{f}(z)/T(z)^2$,

$$H'(z) = \frac{(z-b)\delta\hat{f}(z)}{T(z)^2} - \frac{1-\hat{F}(z)}{T(z)}.$$

Integrating over $[\underline{z}, z_1]$ and letting $z_1 \rightarrow \bar{z}$ shows that output net of b times employment equals $\int_{\underline{z}}^{\bar{z}} (1-\hat{F}(z))/T(z) dz$, because $H(\underline{z}) = 0$ and $H(z_1) \rightarrow 0$. When $\bar{z}$ is finite, the second limit holds because $\hat{F}(\bar{z}) = 1$. When $\bar{z} = \infty$, $|H(z_1)| \leq (z_1 + |b|)(1-\hat{F}(z_1))/\delta$, which converges to zero because output, and hence the mean of $\hat{F}$, is finite. Output net of b times employment therefore equals profits. Hence the wage bill equals b times employment, the average wage of employed workers equals b , and $LS = b \int \ell(z) d\hat{F}(z) / \int z \ell(z) d\hat{F}(z)$. $\square$

A.7 Proof of Proposition 5

The statement for wage posting is Corollary 1. Under sequential auctions, $1 - LS = \mathbb{E}^Y[\tau(z)]/\xi$ because $\sigma = 1/\xi$. Change variables to $t = \kappa(1-\hat{F}(z)) \in (0, \kappa]$. By equation (26), $\tau(t) = (1-\beta)(1+t)/(1+\beta t)$. With Pareto productivity, $z = \underline{z}(t/\kappa)^{-1/\xi}$, and the output weight $z \ell(z) d\hat{F}(z)$ is proportional to $k(t) dt$ with $k(t) \equiv t^{-1/\xi}(1+t)^{-2}$, up to a factor that does not depend on t . Hence

$$\mathbb{E}^Y[\tau(z)] = \frac{\int_0^\kappa \tau(t) k(t) dt}{\int_0^\kappa k(t) dt}, \quad \frac{d \mathbb{E}^Y[\tau(z)]}{d\kappa} = \frac{k(\kappa)(\tau(\kappa) - \mathbb{E}^Y[\tau(z)])}{\int_0^\kappa k(t) dt} > 0,$$

because τ is strictly increasing for $\beta < 1$ and therefore $\mathbb{E}^Y[\tau(z)] < \tau(\kappa)$. As $\kappa \rightarrow 0$, $\mathbb{E}^Y[\tau(z)] \rightarrow \tau(0) = 1-\beta$. As $\kappa \rightarrow \infty$, the integrals converge because $k(t)\tau(t) = O(t^{-1/\xi-1})$ as $t \rightarrow \infty$ and $1/\xi \in (0, 1)$. Let $\sigma = 1/\xi$ and $I_1 \equiv \int_0^\infty t^{-\sigma}(1+t)^{-1} dt$. Since $\tau(t)k(t) = t^{-\sigma}((1+t)^{-1} - \beta(1+\beta t)^{-1})$ and the substitution $x = \beta t$ gives $\int_0^\infty \beta t^{-\sigma}(1+\beta t)^{-1} dt = \beta^\sigma I_1$, the integral of τk over $(0, \infty)$ is $(1-\beta^\sigma)I_1$, which also holds for $\beta = 0$. Since

$$\frac{d}{dt} \frac{t^{1-\sigma}}{1+t} = \frac{t^{-\sigma}}{(1+t)^2} - \sigma \frac{t^{-\sigma}}{1+t}$$

and $t^{1-\sigma}/(1+t)$ vanishes as $t \rightarrow 0$ and as $t \rightarrow \infty$, integrating over $(0, \infty)$ gives $\int_0^\infty k(t) dt = \sigma I_1$. The limit of $\mathbb{E}^Y[\tau(z)]$ is therefore $(1-\beta^\sigma)/\sigma$, and $LS \rightarrow \beta^\sigma = \beta^{1/\xi}$. $\square$

A.8 Proof of Proposition 6

Part (i). Equation (22) gives $1 - \text{LS} = \mathbb{E}^Y[\sigma(z)]$, and hence $\text{LS} = \mathbb{E}^Y[1 - \sigma(z)]$. Since $\varphi(z)/z = 1 - (1 - \hat{F}(z))/(z\hat{f}(z)) = 1 - \sigma(z)$, and $\sigma(z) = 1/\xi(z)$ by equation (13), part (i) follows. The function φ is the marginal revenue along D : with quantity $q = D(p)$, revenue pq satisfies $d(pq)/dq = p + q dp/dq = p - (1 - \hat{F}(p))/\hat{f}(p)$.

Part (ii). By equation (13), $\xi(z) = -d \log D(p)/d \log p$ at $p = z$ is the price elasticity of D ; $\sigma = 1/\xi$ is decreasing if and only if the elasticity is increasing. Let $x = \log z$ and $G(x) \equiv \hat{F}(e^x)$, with density $g(x) = e^x \hat{f}(e^x)$. The hazard rate of x is $g(x)/(1 - G(x)) = z\hat{f}(z)/(1 - \hat{F}(z)) = \xi(z)$. $\square$

A.9 Proof of Proposition 7

Differentiating equation (5) with respect to $\hat{F}$ gives $dL = (1 + \kappa) [1 + \kappa(1 - \hat{F})]^{-2} d\hat{F}$, and equation (4) then gives

$$\ell(z) d\hat{F}(z) = \frac{\lambda^F}{\delta} \frac{dL(z)}{1 + \kappa}, \quad \int_{\underline{z}}^{\bar{z}} z \ell(z) d\hat{F}(z) = \frac{\lambda^F}{\delta} \frac{\mu_L}{1 + \kappa}.$$

By equation (5), $1 + \kappa(1 - \hat{F}) = (1 + \kappa)/(1 + \kappa L)$ and $1 - \hat{F} = (1 - L)/(1 + \kappa L)$. Hence

$$\ell(z)(1 - \hat{F}(z)) = \frac{\lambda^F}{\delta} \frac{(1 - L(z))(1 + \kappa L(z))}{(1 + \kappa)^2}.$$

Wage posting. By Proposition 2, $\pi' = \ell$ and $\pi(\underline{z}) = 0$. Substituting into the expression for $1 - \text{LS}$ in the proof of Lemma 2 gives equation (31).

Sequential auctions. By equation (5), $1 + \beta\kappa(1 - \hat{F}) = (1 + \kappa L + \beta\kappa(1 - L))/(1 + \kappa L)$, and equation (26) becomes $\tau = (1 - \beta)(1 + \kappa)/(1 + \kappa L + \beta\kappa(1 - L))$. Substituting $\pi' = \tau\ell$ and $\pi(\underline{z}) = 0$ into the same expression gives equation (32). $\square$

A.10 Proof of Proposition 8

Let $\psi(z) = \xi z_0^\xi z^{-\xi-1}$ denote the density of Ψ , which satisfies $(z\psi(z))' = -\xi\psi(z)$. Firms with $z < \underline{z}$ post no vacancies, and the integrals below run over $[\underline{z}, \infty)$. Under wage posting, $\pi'(z) = \ell(z)$ (Proposition 2), and $\pi(\underline{z}) = 0$ because $\underline{z} = \underline{w}$. The vacancy condition $c'(v(z)) = \pi(z)$ gives $\pi(z) = \bar{c}v(z)^\omega$. Hence $v(\underline{z}) = 0$, the function $v^{1+\omega} = (\pi/\bar{c})^{1+1/\omega}$ is

continuously differentiable, and

$$z \ell(z) v(z) = z \pi'(z) v(z) = \frac{\bar{c}\omega}{1+\omega} z \frac{d}{dz} (v(z)^{1+\omega}).$$

Integrating aggregate output $Y = \int z \ell v d\Psi$ by parts,

$$Y = \frac{\bar{c}\omega}{1+\omega} \left([z\psi(z) v(z)^{1+\omega}]_{\underline{z}}^{\infty} + \xi \int_{\underline{z}}^{\infty} v(z)^{1+\omega} \psi(z) dz \right).$$

The boundary term vanishes at $\underline{z}$ because $v(\underline{z}) = 0$. At the top, $\ell(z) \leq \lambda^F/\delta$ implies $\pi(z) \leq (z - \underline{z})\lambda^F/\delta$, and hence $v(z) = (\pi(z)/\bar{c})^{1/\omega}$ is of order at most $z^{1/\omega}$. The term $z\psi(z)v(z)^{1+\omega}$ is therefore of order at most $z^{1+1/\omega-\xi}$, which vanishes because $\xi > 1 + 1/\omega$. Aggregate profits are $\int \pi(z) v(z) d\Psi = \bar{c} \int v(z)^{1+\omega} \psi(z) dz$. Hence $1 - \text{LS} = (1+\omega)/(\omega\xi)$. The argument uses neither the meeting and separation rates nor the shape of ℓ . $\square$

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Supplemental Appendix to “Competition for Workers and the Labor Share”

Katsuhiro Komatsu

B Data and Computations for Sections 5 and 6

B.1 Data

The data are public releases of the Census Bureau. The Dispersion Statistics on Productivity (DiSP), produced jointly with the Bureau of Labor Statistics, cover manufacturing in 1987–2021 and retail trade in 1987–2017 ([U.S. Bureau of Labor Statistics and U.S. Census Bureau, 2025](#)). The Business Dynamics Statistics (BDS) are the 2023 release, for 1978–2023 ([U.S. Census Bureau, 2025](#)). Value added by industry in 2013–2019 comes from the Annual Survey of Manufactures (ASM), with the Economic Census in 2017 ([U.S. Census Bureau, 2026](#)).

Productivity distributions. For each four-digit NAICS manufacturing industry and year, the DiSP reports the standard deviation and the differences $x_{75} - x_{25}$, $x_{90} - x_{10}$, $x_{99} - x_{90}$, and $x_{10} - x_1$ of log productivity. It reports them unweighted and weighted by activity, which is hours for output per hour and the composite input of Appendix B.3 for TFP. Cells with data quality concerns are suppressed. For output per hour, the ratio R of equation (37) is available for 3,001 of the 3,010 industry-years in the activity-weighted distributions. In Section 5.3, I weight industries by their employment in the BDS in the same year, and in Sections 5.4 and 5.5, by their value added in the ASM. The DiSP does not report $x_{25} - x_{10}$ and $x_{90} - x_{75}$ separately, only their sum $x_{90} - x_{10} - (x_{75} - x_{25})$. I therefore take the two intervals $[10, 25]$ and $[75, 90]$ together. Their combined length in $-\log(1 - p)$ is $\log 3$, and equation (36) gives their average tail thickness, $(x_{90} - x_{10} - (x_{75} - x_{25}))/\log 3$. At the median, it is 0.96 times $\sigma_{L,[25,75]}$, between the values for a Pareto $\hat{F}$ (0.94) and a lognormal $\hat{F}$ (1.02) at $\kappa = 18.4$. Within industries, R has changed little over time: a regression of R on a linear time trend with industry fixed effects, weighted by employment, gives +0.006 per decade (standard error 0.006, clustered by industry), or about 0.02 over 1987–2021.

B.2 Computations for Section 5.4

Since the DiSP provides only four percentile gaps of log productivity, I interpolate x_p as a function of $-\log(1 - p)$ with a monotone cubic interpolation. The interpolation runs through the percentiles 1, 10, 25, 75, 90, and 99, with $x_1 = 0$ and the remaining gap divided equally (Section 5.4), and through two points on the Pareto line above the 99th percentile. Because the lowest productivity is at the first percentile, the model's L is the distribution above it, $L(\chi \exp(x)) = (L^H(x) - 0.01)/0.99$ for $x \geq x_1$, and the rank in $\hat{F}$ follows from the rank in L by equation (5). Integrals over $-\log(1 - \hat{F}) \in [0, 40]$ use 10,001 points, with Simpson's rule for output and the trapezoid rule in z for profits, and the Pareto tail beyond the grid is integrated analytically. The derivative with respect to $\log \kappa$ is a central difference with steps of ± 0.02 in $\log \kappa$. The threshold β^* is the first value at which the derivative turns positive on a grid of β with step 0.05 on $[0, 0.95]$, refined by root finding.

To check that the numerical integration is accurate, I apply it to distributions whose labor shares I obtain independently. For the Pareto distribution with $\xi = 3$, the computation gives $LS = 2/3$ to five decimals under wage posting for $\kappa = 2.5$ and $\kappa = 10$. For the lognormal distribution of Figure 3, it gives 0.76770 under wage posting and 0.27812 under sequential auctions with $\beta = 0$ at $\kappa = 10$, as does the computation behind the figure. For output per hour, equation (31) agrees with this computation to four decimals in every industry-year.

B.3 TFP-Based Measure

From TFP to z . The DiSP measures TFP as $TFP_j = Q_j / (K_j^{s_K} H_j^{s_H} M_j^{s_M})$, where K_j is capital, M_j is intermediate inputs, and s_K , s_H , and s_M are the shares of capital, labor, and intermediate inputs in the total costs of the establishment's six-digit industry, which sum to one. Since $Q_j/H_j = TFP_j (K_j/H_j)^{s_K} (M_j/H_j)^{s_M}$, the decomposition of Section 5.3 becomes

$$z_j = (1 - \alpha) \times \frac{H_j}{N_j} \times \frac{VA_j}{Q_j} \times \left(\frac{K_j}{H_j}\right)^{s_K} \left(\frac{M_j}{H_j}\right)^{s_M} \times TFP_j,$$

and the DiSP weights TFP by the composite input $K_j^{s_K} H_j^{s_H} M_j^{s_M} = Q_j / TFP_j$. Under the first, second, and fourth conditions of Section 5.3, two polar cases relate the distribution of TFP to L . If capital and intermediate inputs per hour are the same for all establishments of an industry, the fourth factor is common, the composite input is proportional to hours, and log TFP equals log output per hour up to a constant. This case is a benchmark for the scale of

Table B.1: Labor Share and Competition Implied by the DiSP Distributions of TFP

TFP	Derivative with respect to $\log \kappa$						Threshold β^*
	Posting		Sequential auctions				
			$\beta = 0.5$		$\beta = 0$		
	Median	Share > 0	Median	Share > 0	Median	Share > 0	
Unscaled	0.002	0.61	0.000	0.53	-0.060	0.00	0.43
Scaled	0.003	0.57	0.000	0.51	-0.054	0.00	0.44

Notes: As in Table 2, for TFP weighted by composite input. The scaled row excludes industry-years in which the scaled distribution implies infinite output per worker (12 percent of value added).

the gaps only: the share of value added in gross output then rises with output per hour, and the third condition fails. If instead establishments rent capital and buy intermediate inputs freely at common prices, as in Section 5.1, capital and intermediate inputs are proportional to gross output. Because the cost shares sum to one, TFP_j is then proportional to $(Q_j/H_j)^{s_H}$, and z_j to TFP_j^{1/s_H} . The composite input per worker is proportional to $z_j^{1-s_H}$, so that the weights place more weight on productive establishments than L does. The percentile gaps of log output per hour would equal those of log TFP in the first case and be about $1/s_H$ times them in the second, about 5 in manufacturing. They would be exactly $1/s_H$ times them if s_H were common to the industry and both distributions were weighted by hours. In the data for 2013–2019, the interquartile range of log output per hour is 1.7 times that of log TFP at the median. The shapes also differ: the upper tail of TFP is thicker (Section 5.4), and over 1987–2021, with industry fixed effects, R rises by 0.058 per decade for TFP (standard error 0.015). These differences come from what the model leaves out, such as capital that does not adjust to productivity, differences in input prices, and errors in measured inputs.

Results. Table B.1 reports the results of Section 5.4 for TFP in two versions, with the same construction. The first uses the percentile gaps of TFP as they are, as in the first case. The second multiplies them, for each industry and year, by the ratio of the interquartile range of output per hour to that of TFP. This gives TFP the dispersion of output per hour, a scale between those of the two polar cases, and leaves R unchanged. In both versions, the results are weaker than for output per hour (Table 2) because the upper tail is thicker. In 37 and 41 percent of industry-years, respectively, more competition lowers the labor share under sequential auctions even with $\beta = 0.95$.

B.4 Vacancy Creation and the Size of Establishments

Sections 3 to 5 hold $\hat{F}$ fixed. To see how much vacancy creation can change the level of the labor share in U.S. manufacturing, I start from a distribution of productivity across establishments Ψ and compare two cases. With fixed vacancies, every establishment posts the same number of vacancies, and $\hat{F} = \Psi$. With vacancy creation, the curvature ω is chosen to match the employment share of large establishments. Because $\hat{F}$ comes here from Ψ rather than from the distribution of employment as in Section 5, the levels differ from those in Table 1.

The shape of the vacancy-weighted productivity distribution $\hat{F}$ depends on how vacancies vary with productivity, which the curvature ω of Section 6 governs. Given κ , the employment share of large establishments identifies ω . To see that, note that an establishment with productivity z employs $v(z)\ell(z)$ workers, and two forces make productive establishments large: (i) the job ladder, through $\ell(z)$, and (ii) vacancy creation, through $v(z)$. Higher competition κ concentrates employment through the first force, and a lower curvature ω through the second.

To pin down the curvature ω , I use the employment shares of large manufacturing establishments in the BDS for 2019 as targets. In the data, establishments with at least 100, 500, and 1,000 employees make up 9.3, 1.2, and 0.35 percent of all establishments and employ 67, 29, and 16 percent of workers. As the distribution of productivity across establishments $\Psi(z)$, I use a representative manufacturing industry: the unweighted distribution of output per hour, which counts each establishment once, with each percentile gap set to its median across the industry-years of 2013–2019. Given $\Psi(z)$ and ω , I can solve for $\hat{F}$ under wage posting with the meeting rates fixed. I then choose ω so that the same fraction of the most productive establishments employs the same share of workers as in the data.

With $\kappa = 18.4$, the job ladder alone accounts for the employment share of establishments with at least 100 employees: with fixed vacancies, the most productive 9.3 percent of establishments employ 67 percent of workers, as in the data. If I instead match the employment shares of establishments with at least 500 and 1,000 employees, I obtain $\omega = 10.8$ and 6.5, respectively (Table B.2). At their geometric mean, vacancy creation lowers the labor share of value added by 2 percentage points relative to fixed vacancies. This difference depends on the unobserved tail above the 99th percentile. When the distribution is truncated at the 99.9th percentile instead of extended by a Pareto tail, the same shares give similar values

Table B.2: Vacancy-Cost Curvature and the Employment Share of Large Establishments

Upper tail	ω matching the employment share of establishments with at least n employees			Labor share	
	$n = 100$	$n = 500$	$n = 1,000$	Matched ω	Fixed v
Pareto	∞	10.80	6.54	0.42	0.44
Truncated at 99.9%	∞	10.47	6.14	0.47	0.47

Notes: Labor shares of value added under wage posting, with $\alpha = 0.3$. Targets: employment shares of manufacturing establishments with at least 100, 500, and 1,000 employees in the BDS, 2019. ∞ : fixed vacancies already give the target. The labor share at the matched ω uses the geometric mean of the two finite values.

of ω , and vacancy creation changes the labor share by less than 1 percentage point. In both cases, vacancy costs count as part of the non-labor share. If they were counted as labor costs, vacancy creation would raise the labor share.

Details. The percentile gaps of Ψ are those of the unweighted distribution of output per hour, at their medians across the manufacturing industry-years of 2013–2019, weighted by value added. They are $x_{10} - x_1 = 0.578$, $x_{90} - x_{10} = 1.863$, $x_{75} - x_{25} = 0.927$, and $x_{99} - x_{90} = 0.778$. They are joined as in Appendix B.2, with the lowest productivity at the first percentile. Under wage posting with $\underline{z} = \underline{w}$ and fixed meeting rates, vacancies are $v(z) \propto \pi(z)^{1/\omega}$ with $\pi(z) = \int_{\underline{z}}^z \ell(x) dx$, and $\hat{F}(z) = \int_{\underline{z}}^z v d\Psi / \int v d\Psi$. Because ℓ depends on $\hat{F}$ through equation (4), $\hat{F}$ solves a fixed point, which I compute on a grid of the rank in Ψ with a damping weight of one half. Above the 99th percentile, Ψ has a Pareto tail with exponent $\log 10/0.778 = 2.96$; the integrals beyond the grid use the asymptotic Pareto form, and output is finite for $\omega > 1/1.96$. The alternative truncates Ψ at the 99.9th percentile.

C Data and Computations for Sections 5.4 and 5.5

C.1 Flows and Competition

The monthly CPS files for 2013–2019 come from IPUMS (Flood et al., 2025), and job-to-job moves come from the matched records of Fujita et al. (2024a), which impute the missing answers to the employer question as in Fujita et al. (2024b). The population

consists of private wage and salary workers and the unemployed of both sexes aged 22–64. A transition counts only if it starts and ends in this population; in the alternative of Appendix C.2, exits into nonparticipation also count. Pooled over 2013–2019, the monthly probability of separation into unemployment (EU) is 1.09 percent, that of job finding (UE) is 26.2 percent, and that of a job-to-job move is 2.30 percent.

In the stationary job ladder, an employed worker moves when an offer arrives from a firm with a higher rank $\hat{F}(z)$. By equation (5), the mean of $1 - \hat{F}(z)$ over employed workers is $[(1 + \kappa) \log(1 + \kappa) - \kappa] / \kappa^2$, and the job-to-job rate of employed workers is

$$\lambda^E \frac{(1 + \kappa) \log(1 + \kappa) - \kappa}{\kappa^2} = \delta \frac{(1 + \kappa) \log(1 + \kappa) - \kappa}{\kappa}. \quad (\text{C.1})$$

The ratio of the job-to-job hazard to the EU hazard, with each hazard computed as $-\log(1 - p)$ from the monthly probability p , is 2.13, and equation (C.1) gives $\kappa = 18.4$ and an offer arrival rate of 0.20 a month. When separations also include exits into nonparticipation (EN) other than retirement and disability (“retired” and “unable to work” in the CPS), their monthly probability is 2.74 percent, and $\kappa = 2.9$. In the economy with the offer rate of the 1980s in Section 5.5, κ is 1/0.49 times as high. This is the ratio of the values of κ that Engbom et al. (2026, Table 2) estimate for 1982–1991 and 2012–2021, where job changes unrelated to pay count as separations. I attribute the change to the offer arrival rate and keep the separation rate, which matters only for the job-to-job rate given by equation (C.1). Their estimate of κ for 2012–2021, 0.53, is much lower than mine. They infer κ from the gap between the wage distributions of all workers and of hires from nonemployment, and they attribute many job-to-job moves to offers that workers accept regardless of pay. I use only the proportional change.

C.2 Sensitivity

Table C.1 repeats the comparison of Section 5.5 with one assumption changed at a time. The alternatives are the following.

- Separations: exits from employment into unemployment and into nonparticipation, except exits into retirement and disability, which end participation.
- Level of competition: $\kappa = 0.53$, the estimate of Engbom et al. (2026) for 2012–2021

Table C.1: Sensitivity of the Effect of the Decline in Outside Offers

	κ	Labor share, 2013–2019			Decline in offers since the 1980s: change in the labor share (p.p.)		
		Posting	Sequential auctions		Posting	Sequential auctions	
			$\beta = 0.5$	$\beta = 0$		$\beta = 0.5$	$\beta = 0$
Baseline	18.4	0.42	0.53	0.12	−1.8	−0.9	+2.5
Separations EU + EN	2.9	0.35	0.49	0.12	−4.0	−1.8	+2.1
Level of κ in Engbom et al. (2026)	0.53	0.23	0.44	0.12	−5.1	−1.6	+1.2
Percentile gaps $\times 0.9$	18.4	0.44	0.54	0.15	−1.7	−0.8	+2.8
Percentile gaps $\times 0.65$	18.4	0.51	0.58	0.23	−1.4	−0.7	+3.4
Share of the gap in $[10, 25]$: 0.3	18.4	0.40	0.52	0.13	−1.0	−0.3	+3.0
Share of the gap in $[10, 25]$: 0.7	18.4	0.44	0.53	0.11	−2.8	−1.6	+2.0
Decline in offers of 33%	18.4	0.42	0.53	0.12	−1.0	−0.5	+1.5

Notes: Labor share: average of the labor shares of value added over the industry-years of 2013–2019, weighted by industry value added in the ASM, with $\hat{F}$ recovered at the value of κ in the second column. Decline in offers since the 1980s: change from the economy with the offer rate of the 1980s in Section 5.5, in percentage points, with $\hat{F}$ fixed.

(Appendix C.1).

- Persistent dispersion: every percentile gap multiplied by 0.9 or by 0.65. If the rest of the dispersion is transitory, 0.9 corresponds to the autocorrelation of 0.81 in log labor productivity that [Gouin-Bonenfant \(2022\)](#) reports for Canadian firms, and 0.65 to an autocorrelation of 0.42.
- Remaining gap: 30 or 70 percent of $x_{90} - x_{10} - (x_{75} - x_{25})$ lies in $[10, 25]$, instead of one half.
- Decline in offers: 33 percent instead of 51 percent.

The decline in offers lowers the labor share in every specification under wage posting and under sequential auctions with $\beta = 0.5$, and raises it with $\beta = 0$. Under wage posting and sequential auctions with $\beta = 0.5$, the effects are largest when competition is much lower, with exits into nonparticipation counted as separations or with the level of κ of [Engbom et al. \(2026\)](#). For the same decline in offers, they are smallest when $[10, 25]$ holds 30 percent of the remaining gap.

D Additional Proofs

This appendix proves Lemma 1, derives the profit multiplier under continuously renegotiated Nash bargaining, gives the parts of the proofs of Propositions 2 and 4 that restate results of the literature, and treats sequential auctions with discounting.

D.1 Proof of Lemma 1

Let $L(z)$ denote, as in Section 2.3, the share of employed workers who work at firms with productivity at most z , so that these firms employ $(1 - u)L(z)$ workers. Flows into this set come from unemployed workers who meet an active firm with productivity at most z , at rate $\lambda^U u \hat{F}(z)$. Flows out of it come from separations, at rate $\delta(1 - u)L(z)$, and from moves to more productive firms, at rate $\lambda^E(1 - \hat{F}(z))(1 - u)L(z)$. Moves between two firms in the set do not change $L(z)$. In steady state,

$$\lambda^U u \hat{F}(z) = T(z)(1 - u)L(z). \quad (\text{D.1})$$

At $z = \bar{z}$, $L(\bar{z}) = 1$ and $T(\bar{z}) = \delta$, which gives $\delta(1 - u) = \lambda^U u$ and $u = \delta/(\delta + \lambda^U)$.

Because a vacancy meets a worker drawn in proportion to search intensity, the probability that the worker is unemployed or employed by a firm with productivity at most z is

$$\hat{G}(z) = \frac{\lambda^U u + \lambda^E(1 - u)L(z)}{\lambda^U u + \lambda^E(1 - u)}.$$

By equation (D.1), the numerator equals $\lambda^U u [1 + \lambda^E \hat{F}(z)/T(z)] = \lambda^U u (\delta + \lambda^E)/T(z)$. Since $\lambda^U u = \delta(1 - u)$, the denominator equals $\lambda^U u (1 + \lambda^E/\delta) = \lambda^U u (\delta + \lambda^E)/\delta$. Hence $\hat{G}(z) = \delta/T(z)$.

A vacancy of a firm with productivity z meets workers at rate λ^F and hires the worker it meets whenever the worker is unemployed or employed by a less productive firm. This gives $h(z) = \lambda^F \hat{G}(z)$. Each worker of the firm separates or moves to a more productive firm at rate $T(z)$. In steady state, the hires of a vacancy equal the workers who leave it, $h(z) = T(z) \ell(z)$, and hence $\ell(z) = h(z)/T(z)$. Substituting $\hat{G} = \delta/T$ gives the remaining expressions in equation (4). Integrating over vacancies, employment is $V \int \ell d\hat{F} = V\lambda^F/(\delta + \lambda^E)$, which equals $1 - u$ because the meetings of vacancies and of workers coincide, $V\lambda^F = \lambda^U u + \lambda^E(1 - u)$. $\square$

D.2 Continuously Renegotiated Nash Bargaining

This subsection derives the profit multiplier in footnote 9. The wage is renegotiated continuously, and the worker receives the share $\beta \in (0, 1)$ of the match surplus, with unemployment as the disagreement point (Gottfries, 2026). The values are derived for $r > 0$, and the limit $r \rightarrow 0$ is taken at the end. Let U denote the value of unemployment, $W(z)$ the value of a worker employed by a firm with productivity z , and $J(z)$ the value of the worker to the firm. The match surplus $S(z) \equiv J(z) + W(z) - U$, which differs from the surplus under sequential auctions in Proposition 4, is divided as

$$W(z) - U = \beta S(z), \quad J(z) = (1 - \beta) S(z).$$

Suppose that S is strictly increasing, which is verified below. Then a worker moves to another firm if and only if it is more productive than the current employer, part (i) of Assumption 1 holds, and the firm loses the worker at rate $T(z)$ in equation (2). A worker who moves to a firm with productivity $z'' > z$ gains $\beta(S(z'') - S(z))$. Hence

$$(r + T(z)) J(z) = z - w(z),$$

$$r W(z) = w(z) + \delta(U - W(z)) + \beta \lambda^E \int_z^{\bar{z}} (S(z'') - S(z)) d\hat{F}(z'').$$

Adding the two equations, subtracting rU , and using the division of the surplus gives

$$(r + T(z)) S(z) = z - rU + \beta \lambda^E \int_z^{\bar{z}} S(z'') d\hat{F}(z''),$$

and differentiating with respect to z gives

$$S'(z) = \frac{1 + (1 - \beta) \lambda^E \hat{f}(z) S(z)}{r + T(z)}.$$

An unemployed worker accepts every offer with $S \geq 0$, and the reservation level is the productivity at which the surplus vanishes, $S(\underline{w}) = 0$. Since $S' > 0$ wherever $S \geq 0$, S is positive and strictly increasing above $\underline{w}$, as conjectured. As under the two protocols, $\underline{z} = \underline{w}$.

A vacancy hires $h(z) = \lambda^F \hat{G}(z) = \lambda^F \delta / T(z)$ workers per unit of time by Lemma 1, and

in the limit $r \rightarrow 0$, $J(z) = (z - w(z))/T(z)$. Since $\ell = h/T$, profits per vacancy are

$$\pi(z) = (z - w(z)) \ell(z) = h(z) J(z) = (1 - \beta) h(z) S(z).$$

With $h'(z)/h(z) = -T'(z)/T(z) = \lambda^E \hat{f}(z)/T(z)$ and the limit of S' as $r \rightarrow 0$,

$$\pi'(z) = (1 - \beta) [h(z) S'(z) + h'(z) S(z)] = (1 - \beta) \ell(z) \left[1 + (2 - \beta) \lambda^E \hat{f}(z) S(z) \right],$$

which gives $\tau(z)$ in footnote 9. Since $S(\underline{z}) = 0$, the least productive active firm earns $\pi(\underline{z}) = 0$. $\square$

D.3 Proof of Proposition 2: Part (i) and the Reservation Wage

Let F and $\tilde{\ell}$ be as in Appendix A.3.

Part (i). Three properties follow from the optimality of posted wages. First, F has no mass point. If a positive mass of vacancies posted w , a vacancy posting w could raise its wage slightly and also hire the workers employed at w , a discrete gain in employment at an arbitrarily small cost per worker. Second, the support of F has no gaps. If no vacancy posted a wage in (w_1, w_2) while some posted w_2 , those vacancies could lower their wage toward w_1 without losing workers, because $\tilde{\ell}$ is constant on the gap. Third, the optimal wage is nondecreasing in z . If w is optimal for z and w' for $z' > z$, adding the two optimality conditions gives $(z' - z)(\tilde{\ell}(w') - \tilde{\ell}(w)) \geq 0$, and $w' \geq w$ because $\tilde{\ell}$ is strictly increasing on the support. A flat part of $w(z)$ would make a positive mass of firms post the same wage, which the first property rules out. The optimal wage is therefore strictly increasing in z , and it is continuous because the support has no gaps. Hence $F(w(z)) = \hat{F}(z)$, workers move to firms that pay more, which are the more productive firms, and part (i) of Assumption 1 holds. The least productive active firm hires only from unemployment, and raising its wage above the lowest acceptable wage does not raise its employment. It therefore pays the reservation wage $\underline{w}$.

Reservation wage. Let U denote the value of unemployment and $W(w)$ the value of a job that pays w , for a worker who discounts at rate r , and let $\bar{w} = w(\bar{z})$ denote the highest wage. An unemployed worker accepts every offer, and an employed worker accepts every

offer that pays more. Hence

$$rU = b + \lambda^U \int_{\underline{w}}^{\bar{w}} (W(x) - U) dF(x),$$

$$rW(w) = w + \delta(U - W(w)) + \lambda^E \int_w^{\bar{w}} (W(x) - W(w)) dF(x).$$

Differentiating the second equation, and using that F has no mass points, gives $W'(w) = 1/(r + \delta + \lambda^E(1 - F(w)))$. The reservation wage satisfies $W(\underline{w}) = U$. Subtracting the first equation from the second at $w = \underline{w}$ gives $\underline{w} - b = (\lambda^U - \lambda^E) \int_{\underline{w}}^{\bar{w}} (W(x) - U) dF(x)$. Integrating by parts, with $W(\underline{w}) - U = 0$ and $F(\bar{w}) = 1$ (when $\bar{w} = \infty$, the boundary term $(W(w) - U)(1 - F(w))$ vanishes as $w \rightarrow \infty$, because $W' \leq 1/(r + \delta)$ and the mean of F is finite, as wages lie below productivity and output is finite),

$$\underline{w} - b = (\lambda^U - \lambda^E) \int_{\underline{w}}^{\bar{w}} (1 - F(w)) W'(w) dw = (\lambda^U - \lambda^E) \int_{\underline{w}}^{\bar{w}} \frac{1 - F(w)}{r + \delta + \lambda^E(1 - F(w))} dw,$$

whose limit as $r \rightarrow 0$ is equation (20). $\square$

D.4 Proof of Proposition 4: Parts (i) and (ii)

The values are derived for $r > 0$, and the limit $r \rightarrow 0$ is taken at the end. Let U denote the value of unemployment. Let $S(z)$ denote the value, in excess of U , of a worker employed by a firm with productivity z who receives the entire surplus of the match, that is, the maximal offer of the firm.

Part (i). Consider a worker employed by a firm with productivity z . If the worker meets a more productive firm with productivity z'' , the worker moves and obtains $S(z) + \beta(S(z'') - S(z))$, and the current employer obtains nothing. A meeting with a less productive firm changes only the division of the surplus between the worker and the current employer. Hence the joint value of the worker and the current employer, in excess of U , does not depend on the current contract. Denote it by X . It satisfies

$$(r + \delta) X = z - rU + \lambda^E \int_z^{\bar{z}} [S(z) + \beta(S(z'') - S(z)) - X] d\hat{F}(z'').$$

The joint value equals the value of the worker who receives the entire surplus, $X = S(z)$,

which gives

$$(r + \delta) S(z) = z - rU + \beta \lambda^E \int_z^{\bar{z}} (S(z'') - S(z)) d\hat{F}(z''). \quad (\text{D.2})$$

Differentiating with respect to z gives $(r + \delta + \beta \lambda^E (1 - \hat{F}(z))) S'(z) = 1$, whose limit as $r \rightarrow 0$ is equation (23). An unemployed worker hired by a firm with productivity z'' obtains $\beta S(z'')$, which gives $rU = b + \beta \lambda^U \int S(z) d\hat{F}(z)$. The reservation level is the productivity at which the surplus vanishes, $S(\underline{w}) = 0$. Evaluating equation (D.2) at $z = \underline{w} = \underline{z}$, where $\hat{F}(\underline{w}) = 0$, gives $0 = \underline{w} - rU + \beta \lambda^E \int S(z) d\hat{F}$. Combining the two conditions gives equation (24). Since S is strictly increasing, workers move to more productive firms and accept offers from every firm with $z > \underline{w}$, and part (i) of Assumption 1 holds.

Part (ii). At the time of hiring, a worker whose competing offer comes from a firm with productivity $z' < z$ obtains $S(z') + \beta(S(z) - S(z'))$, and the firm obtains the rest of the joint value $S(z)$. This gives equation (25). Because the joint value equals $S(z)$ under every division of the surplus, $J(z', z)$ already accounts for the later renegotiations and for the loss of the worker to a more productive firm. For a worker hired from unemployment, $z' = \underline{w}$ and $S(\underline{w}) = 0$. $\square$

D.5 Sequential Auctions with Discounting

Proposition D.1 (Sequential auctions with discounting). *Let the discount rate be $r \geq 0$. Under sequential auctions with $\beta \in [0, 1)$, Proposition 4 holds with $r + \delta$ in place of δ in equation (23) and with*

$$\tau(z) = (1 - \beta) \frac{1 + \kappa_r (1 - \hat{F}(z))}{1 + \beta \kappa_r (1 - \hat{F}(z))}, \quad \kappa_r \equiv \frac{\lambda^E}{r + \delta}, \quad (\text{D.3})$$

in place of equation (26). The allocation, and hence ℓ , depends on $\kappa = \lambda^E / \delta$ as before.

The proof computes the steady-state profits of a vacancy from the profit flows of the workers it currently employs, which differ from the values $J(z', z)$ of its hires when $r > 0$. The profit multiplier τ compares the horizon for which the firm's productivity counts in the joint value with the time the worker spends at the firm. With discounting, both lengths of time are discounted at rate r , which shortens the longer one, the horizon of the joint value,

proportionally more. As r grows, τ approaches $1 - \beta$ at every firm.

Proof. Let $\rho \equiv r + \delta$ and $q(x) \equiv 1 - \hat{F}(x)$. The proof has three steps. The first derives the profit flow of a match from the values of the worker and of the match. The second shows that the number of workers of a vacancy whose last competing offer is at most a given level does not depend on the productivity of the vacancy, as long as this productivity exceeds the level. The third differentiates the profits of a vacancy with respect to productivity.

Values and wages. Values are measured in excess of the value of unemployment U , so that, unlike in Appendices D.2 and D.3, W denotes the value of the worker minus U . As in the proof of Proposition 4(i),

$$\rho S(z) = z - rU + \beta\lambda^E \int_z^{\bar{z}} (S(x) - S(z)) d\hat{F}(x),$$

which gives $S'(z) = 1/(\rho + \beta\lambda^E q(z))$, with $S(\underline{w}) = 0$. Consider a worker employed by a firm with productivity z whose last competing offer came from a firm with productivity $z' < z$. The worker earns the wage $w(z', z)$ and has the value $W(z', z) = (1 - \beta)S(z') + \beta S(z)$. If the worker meets a firm with productivity $x > z$, the worker moves and obtains $(1 - \beta)S(z) + \beta S(x)$. If $z' < x < z$, the worker stays and obtains $(1 - \beta)S(x) + \beta S(z)$. If $x < z'$, this falls short of the current value, and nothing changes (Section 2.2). The value of the worker therefore satisfies

$$\begin{aligned} \rho W(z', z) = w(z', z) - rU + \lambda^E \int_z^{\bar{z}} [(1 - \beta)S(z) + \beta S(x) - W(z', z)] d\hat{F}(x) \\ + \lambda^E \int_{z'}^z [(1 - \beta)S(x) + \beta S(z) - W(z', z)] d\hat{F}(x). \end{aligned}$$

Subtracting it from the equation for S and using $S(z) - W(z', z) = (1 - \beta)(S(z) - S(z'))$ gives the profit flow of the match,

$$z - w(z', z) = (1 - \beta) \left[(\rho + \lambda^E q(z)) (S(z) - S(z')) + \lambda^E \int_{z'}^z (S(x) - S(z')) d\hat{F}(x) \right], \quad (\text{D.4})$$

which vanishes at $z' = z$. When z rises with z' fixed, the terms in $\hat{f}(z)$ cancel, and

$$\frac{\partial [z - w(z', z)]}{\partial z} = (1 - \beta) (\rho + \lambda^E q(z)) S'(z) = (1 - \beta) \frac{\rho + \lambda^E q(z)}{\rho + \beta\lambda^E q(z)},$$

which does not depend on z' . Since equation (D.4) vanishes at $z' = z$, integrating gives $z - w(z', z) = \int_{z'}^z \tau(x) dx$, with τ in equation (D.3). A higher τ therefore lowers every wage, including the starting wage $w(\underline{w}, z)$ of a worker hired from unemployment. With $\beta = 0$, $\underline{w} = b$, and the starting wage is $b - \kappa_r \int_b^z (1 - \hat{F}(x)) dx < b$.

Composition of the workforce. Consider the workers of a vacancy with productivity z whose last competing offer came from a firm with productivity at most $z' < z$, including the workers hired from unemployment, whose competing offer is $\underline{w}$. Workers enter this group when the vacancy hires them, at rate $\lambda^F \hat{G}(z')$. They leave it when they separate, at rate δ , and when they meet a firm more productive than z' , at rate $\lambda^E q(z')$: they then either move to that firm or obtain a new competing offer above z' . None of these rates depends on z , and in the steady state the size of the group is

$$\frac{\lambda^F \hat{G}(z')}{\delta + \lambda^E q(z')} = \ell(z')$$

for every $z > z'$. The equality holds because the same balance determines the employment of a vacancy with productivity $z' \geq \underline{z}$.

Profit multiplier. The profit flow of a vacancy adds up the profit flows of its workers in equation (D.4),

$$\pi(z) = \int_{[\underline{w}, z)} (z - w(z', z)) d\ell(z'),$$

where the measure $d\ell$ has a mass $\ell(\underline{z})$ at $\underline{w} = \underline{z}$ and the density ℓ' on $(\underline{z}, z)$. By the previous step, this measure does not depend on z , and the integrand vanishes at $z' = z$. Differentiating with respect to z therefore gives

$$\pi'(z) = \int_{[\underline{w}, z)} \frac{\partial [z - w(z', z)]}{\partial z} d\ell(z') = (1 - \beta)(\rho + \lambda^E q(z)) S'(z) \ell(z),$$

and

$$\tau(z) = \frac{\pi'(z)}{\ell(z)} = (1 - \beta) \frac{\rho + \lambda^E q(z)}{\rho + \beta \lambda^E q(z)},$$

which is equation (26) with $\kappa_r = \lambda^E / \rho$ in place of κ . A vacancy with productivity $\underline{z} = \underline{w}$ employs only workers whose competing offer is $\underline{w}$, and equation (D.4) gives $\pi(\underline{z}) = 0$. The reservation level follows from $S(\underline{w}) = 0$ as in Proposition 4, with ρ in place of δ , and the labor share follows from Lemma 2. This proves Proposition D.1. When competition

rises through λ^E with r and δ fixed, $\kappa_r/\kappa = \delta/(r + \delta)$ stays constant, and the argument for monotonicity in the proof of Proposition 5 applies with $\tau(\delta t/(r + \delta))$ in place of $\tau(t)$. $\square$

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