

Wealth, Search, and the Job Ladder ^{*}

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Abstract

This paper studies how the wealth accumulation of workers and their job search behavior influence firm wage setting. I incorporate wealth accumulation and endogenous search intensity of risk-averse workers into an equilibrium job-ladder model. The wage-sensitive search of wealth-poor workers restrains wage markdowns near the bottom of the job ladder. Removing this discipline through complete markets allows low-wage firms to further push down wages, lowering the average wage and raising wage inequality. This firm-side response, which I call the *insurance channel*, offsets the direct consumption-smoothing gain of unemployment insurance.

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1 Introduction

How hard workers search for better-paying jobs depends on their wealth. Unemployed workers with little wealth get reemployed faster, and employed workers with little wealth search harder to climb the job ladder (Chetty 2008, Griffy 2021, Kaas et al. 2026). These search decisions, in turn, discipline the wage-setting power of firms. An employer whose employees aggressively pursue outside offers cannot push wages down far below productivity without losing its workforce. Together, the two observations suggest that the wealth of workers should shape the wages firms post, and the wage structure should in turn shape how workers save. Any policy that substitutes for private wealth, such as unemployment insurance, then reaches beyond household budgets into the wages set by firms.

Existing models capture one side of this interaction at a time. In search models with incomplete markets, wealth shapes workers’ search behavior, but wages are taken as given or bargained match by match with an exogenous split of the surplus (Lentz and Tranaes 2005, Lise 2013, Kaas et al. 2026). In equilibrium job-ladder and monopsony models, worker mobility disciplines posted wages, but they abstract from wealth accumulation (Burdett and Mortensen 1998, Bontemps et al. 2000, Gouin-Bonenfant 2022). This paper links the two, shows that the wealth distribution is a key input into firms’ wage-setting power, and quantifies its importance for the wage structure and for the labor-market impact of social insurance policy.

To formalize and quantify this link, I extend the canonical equilibrium job-ladder model of Burdett and Mortensen (1998) with firm-side productivity heterogeneity by incorporating wealth accumulation and endogenous search intensity among risk-averse workers. Workers accumulate precautionary savings as they climb the ladder and run them down after job loss. How intensely they search, off and on the job, depends on their wealth through the marginal utility of consumption. On the firm side, heterogeneous producers decide how many vacancies to create and what wage to post, jointly shaping the ladder that workers climb. The wealth distribution of job seekers along the ladder determines each firm’s vacancy yield and turnover rate, and consequently the entire posted wage schedule.

Firm-side wage setting is summarized by the standard monopsony wage equation. A firm

with productivity y posts a wage equal to

$$w(y) = y - \frac{1}{\epsilon(y)}, \quad (1)$$

where $\epsilon(y)$ is the semi-elasticity of firm size, or firm-level labor supply, with respect to the wage.¹ The formula itself commonly appears in the [Burdett and Mortensen \(1998\)](#) framework, where the semi-elasticity is pinned down by the search friction and the productivity distribution of firms. What is special in my framework is what stands behind $\epsilon(y)$: the semi-elasticity depends on the job search behavior of workers, and hence on their wealth holdings.

To make this dependence transparent, I analytically decompose the labor supply semi-elasticity $\epsilon(y)$ into a poaching margin and a retention margin, and the retention margin further into the local density of competing wage offers and the wage sensitivity of employees' search. The last component is where wealth particularly matters. The semi-elasticity of individual search with respect to the wage declines with wealth in the calibrated model, reflecting the high marginal utility of consumption of the wealth-poor. Since wealth-poor workers are concentrated on the bottom rungs of the job ladder, their wage-sensitive search disciplines the wage-setting behavior of low-wage firms.

I calibrate the model using U.S. data, targeting labor-market flows, the distribution of firm size, the dispersion of firm wage effects, workers' median liquid wealth, and the search behavior of employed workers. The calibrated model reproduces salient untargeted features of the joint distribution of wealth, wages, and mobility.

I first use the calibrated model to ask how much of the equilibrium wage structure originates from market incompleteness. To that end, I compare the baseline economy with two extreme benchmarks, holding every parameter fixed. The first benchmark is the perfect-insurance (or complete-markets) benchmark, where I introduce a representative family that pools income so that individual wealth becomes irrelevant for job search decisions. The second benchmark is the hand-to-mouth benchmark, where workers consume their income each instant and face job-loss risk without any self-insurance. The two benchmarks correspond to perfect insurance and zero insurance against job-loss risk, and the calibrated economy lies between them.

¹Or, equivalently, $w(y) = \frac{\phi(y)}{1+\phi(y)}y$, where $\phi(y)$ is the elasticity of firm size with respect to the wage.

I find that wealth restrains wage-setting power near the bottom of the job ladder. Moving from the baseline to perfect insurance removes the precautionary motive behind low-wage workers' search. Search intensity falls strongly at the bottom rungs, and it becomes less wage-sensitive. In turn, low-productivity firms cut wage offers by up to 1.3%. In the aggregate, perfect insurance lowers the average wage by 1.1%, raises the standard deviation of log wages by 6.4%, and raises the unemployment rate by 1.2 percentage points (p.p.). In response to improved profitability, low-productivity firms also expand by posting more vacancies, reducing the average labor productivity by 0.6%. In contrast, the hand-to-mouth economy moves essentially every outcome in the opposite direction.

Building on this mechanism, I evaluate the equilibrium impact of unemployment insurance (UI), the leading policy substitute for private wealth. I raise the UI replacement rate from 40% in the baseline to 50%. I then decompose the labor-market response into a conventional disincentive channel (or a moral hazard channel), which operates even under perfect insurance, and what I call the *insurance channel*, which operates only because markets are incomplete. The two channels matter for different outcomes. The response of unemployment to UI is almost entirely driven by the disincentive channel. In contrast, the responses of the average wage and of wage dispersion are overstated by 50 to 60% when the insurance channel is ignored.

The insurance channel also changes the welfare assessment of UI. Even under perfect insurance, the policy raises welfare, as the higher reservation wage pushes the least productive firms out of the market and reallocates employment toward more productive firms. Perhaps surprisingly, the welfare gain is smaller in the baseline incomplete-markets economy, the economy in which the policy provides actual insurance. The reason is the firm-side response specific to incomplete markets. The insurance channel weakens the search discipline of wealth-poor workers, low-productivity firms create more vacancies in response to improved profitability and retain workers who would otherwise reallocate toward more productive firms, and the resulting efficiency loss offsets the consumption-smoothing gain.

Overall, I find that the workers' wealth distribution is a quantitatively important equilibrium input into firm wage setting, and self-insurance through private wealth restrains the wage-setting power of low-wage employers. Any government policy that insures workers against risks could inherently affect the wage-setting power of firms and equilibrium labor-market outcomes through the insurance channel even without directly distorting incentives.

Related literature. The first strand is the literature that studies job search and labor-market outcomes under incomplete markets. Partial-equilibrium models show how wealth shapes the search behavior of risk-averse workers ([Lentz and Tranaes 2005](#), [Lise 2013](#)). A growing number of papers also incorporate wealth accumulation in equilibrium search models ([Krusell et al. 2010](#), [Herkenhoff 2019](#), [Braxton et al. 2024](#), [Eeckhout and Sepahsalari 2024](#), [Herkenhoff et al. 2024](#), [Huang and Qiu 2026](#)), mainly focusing on unemployed workers. [Griffy \(2021\)](#) and [Chaumont and Shi \(2022\)](#) combine on-the-job search with savings in a competitive search framework, but without firm-side productivity heterogeneity. I contribute to the literature by incorporating wealth accumulation into the canonical random-search job-ladder model of [Burdett and Mortensen \(1998\)](#) with productivity heterogeneity, which allows me to study how the wealth accumulation of workers matters for between-firm competition over employed workers.

The closest paper is [Kaas et al. \(2026\)](#), who embed a job ladder into an incomplete-markets overlapping-generations general equilibrium model to study tax and transfer policies. The central difference is the wage-setting protocol. [Kaas et al. \(2026\)](#) consider a sequential auction in the spirit of [Cahuc et al. \(2006\)](#). Under wage bargaining, wealth operates worker by worker, and assets shift individual outside options and renegotiated wages. Under wage posting in my paper, firms cannot condition on individual wealth, and wealth operates through the market. The entire wealth distribution of job seekers determines vacancy yields and turnover along the ladder and thereby disciplines the entire posted schedule, and the mapping from the wealth distribution to wage-setting power is the object of interest in this paper. Moreover, their firms differ in match-specific quality rather than permanent productivity, and therefore their framework is silent on how wealth reshapes competition between high- and low-productivity employers. In practice, wage setting likely combines elements of posting and bargaining, and the two papers are complementary in studying the role of wealth accumulation in the labor-market equilibrium with on-the-job search.

The second strand is the literature on the wage-setting power of firms under search frictions ([Burdett and Mortensen 1998](#), [Bontemps et al. 2000](#), [Fukui 2020](#), [Engbom and Moser 2022](#), [Gouin-Bonenfant 2022](#), [Berger et al. 2024](#), [Jarosch et al. 2024](#), [Bilal and Lhuillier 2025](#), [Lindenlaub et al. 2024](#), [Lhuillier 2025](#)), in which worker mobility determines wages without wealth accumulation on the worker side. In contrast to these papers, I demonstrate the role of workers' wealth in shaping wage-setting power by incorporating wealth accumulation

among risk-averse workers. A parallel monopsony literature generates wage-setting power not from search frictions but from job differentiation through idiosyncratic taste shocks. In particular, [Berger et al. \(2026\)](#) and [Volpe \(2026\)](#) study wage-setting power and its implications in a static framework where wage markdowns depend on the composition of a firm’s workforce due to heterogeneous preferences for higher wages. Their composition channel operates in my framework too, but comes from different sources. In contrast to their static framework, the workforce composition, or the wealth distribution along the job ladder, in my framework is itself an equilibrium outcome determined by workers’ dynamic consumption-saving decisions, and it disciplines firms through the search-intensity and turnover margins rather than through taste-based choice elasticities.

The third strand analyzes the aggregate labor-market effects of unemployment insurance ([Mitman and Rabinovich 2015](#), [Landais et al. 2018](#), [Chodorow-Reich et al. 2019](#), [Birinci and See 2023](#), [Karahan et al. 2025](#)). This literature emphasizes the disincentive effects of unemployment insurance that operate through unemployed workers; the improved unemployment value changes the job search behavior of unemployed workers (i.e., moral hazard), and the improved outside option of workers discourages the job creation of firms. Instead, this paper emphasizes the insurance channel that operates through the job search behavior of employed workers and demonstrates its quantitative importance for the equilibrium wage structure. The insurance channel in this paper is somewhat related to [Acemoglu and Shimer \(1999\)](#), who show that the insurance provided by UI can improve allocation by letting risk-averse unemployed workers pursue high-productivity jobs. My insurance channel operates in the opposite direction: by relaxing the search discipline of wealth-poor employed workers and through the induced firm-side response, it weakens reallocation toward productive firms in a job-ladder setting.

On the technical side, the worker block builds on the continuous-time methods of [Achdou et al. \(2022\)](#), which [Huang and Qiu \(2026\)](#) also apply to an equilibrium search model with bargaining. This paper applies the approach to a wage-posting equilibrium.

2 Model

I develop an equilibrium search model that extends [Burdett and Mortensen \(1998\)](#) by introducing borrowing and saving and an endogenous search-intensity choice by risk-averse

workers. The worker-side problem is built on the partial-equilibrium search literature (Lise, 2013). The model is set in continuous time with an infinite horizon. Throughout the paper, I focus on the steady state.

2.1 Environment

2.1.1 Workers

There is a unit mass of homogeneous, infinitely lived, risk-averse workers who discount the future at rate ρ . A worker is either employed at a single firm at wage w or unemployed with non-labor income b . Utility from consumption c is given by $u(c)$, with $u'(c) > 0$ and $u''(c) < 0$ for $c > 0$.

Workers can save and borrow at interest rate r , subject to a borrowing constraint $a \geq \underline{a}$. Wealth a evolves according to

$$\dot{a} = \begin{cases} ra + w - c & \text{if employed,} \\ ra + b - c & \text{if unemployed,} \end{cases} \quad (2)$$

where $\dot{a}$ is the derivative of a with respect to time. Both employed and unemployed workers engage in job search. A worker chooses search intensity $s \geq 0$, incurring flow disutility $\psi(s)$, with $\psi'(s) > 0$ and $\psi''(s) > 0$ for $s > 0$. The unit of search intensity is normalized so that a worker who exerts intensity s contacts vacancies at rate $s\lambda$, where λ is the equilibrium worker-side contact rate determined by the aggregate matching technology described later in Section 2.1.3. Search is random, and conditional on contact, the worker receives a wage offer w drawn from a distribution $F(w)$ supported on $[\underline{w}, \bar{w}]$. Both λ and F are equilibrium objects that the individual worker takes as given. An ongoing match is destroyed exogenously at rate δ and endogenously when the worker accepts a better outside offer.

2.1.2 Firms

There is a fixed measure N of firms that differ in productivity y , distributed according to $\Gamma(y)$ over $[b, \bar{y}]$.² Productivity y is fixed over time. Firms operate constant-returns-to-scale

²Alternatively, I can assume that the support of $\Gamma(y)$ is $[\underline{y}, \bar{y}]$ with $\underline{y} < b$. But as it turns out, firms with $y \leq b$ cannot profitably attract workers and cannot be active in the labor market.

technologies producing output y per worker and discount the future at rate ρ_f . There is no entry or exit.

A firm chooses a single wage w and a vacancy mass v at any point in time. It commits to pay w to every worker hired through that vacancy posting for the entire duration of the match. The wage cannot be renegotiated, made contingent on the worker's wealth or tenure, or revised in response to outside offers. Creating v vacancies costs $c(v)$, where the vacancy-cost function $c(\cdot)$ is strictly increasing and convex.³ Each vacancy contacts workers at rate q , determined by the aggregate matching technology and taken as given by the firm.

2.1.3 Aggregate Matching

The aggregate flow of new matches is governed by a constant-returns-to-scale matching function $\mathcal{M}(S, V)$, where S is aggregate search intensity and V is aggregate vacancies. I define labor-market tightness as $\theta = V/S$. The worker-side contact rate per unit of search intensity is $\lambda = \mathcal{M}(1, \theta)$ and a worker with intensity s meets vacancies at rate $s\lambda$. The vacancy filling rate is $q = \mathcal{M}(\theta^{-1}, 1)$ and constant returns to scale yield $\lambda = \theta q$. The aggregates S and V , and therefore λ and q , are determined in equilibrium.

2.2 Worker Problem

Throughout this subsection, an individual worker takes the wage-offer distribution F and the worker-side contact rate λ as given.

Value functions. Let $U(a)$ denote the value of unemployment for a worker with wealth a , and $W(a, w)$ the value of employment at wage w with wealth a . They satisfy the following Hamilton–Jacobi–Bellman (HJB) equations:

$$\begin{aligned} \rho U(a) = & \max_{c, s} u(c) - \psi(s) + U'(a) (ra + b - c) \\ & + s\lambda \int_{\underline{w}}^{\bar{w}} \max\{W(a, w) - U(a), 0\} dF(w), \end{aligned} \tag{3}$$

³The symbol c also denotes consumption on the worker side; the argument disambiguates the two uses.

$$\begin{aligned} \rho W(a, w) = & \max_{c, s} u(c) - \psi(s) + \partial_a W(a, w) (ra + w - c) \\ & + s\lambda \int_{\underline{w}}^{\bar{w}} \max\{W(a, w') - W(a, w), 0\} dF(w') + \delta (U(a) - W(a, w)), \end{aligned} \quad (4)$$

subject to the borrowing constraint $a \geq \underline{a}$, where ∂_a is shorthand notation for $\partial/\partial a$.

The first three terms on the right-hand side of each HJB equation capture flow utility from consumption, the disutility of search, and the evolution of wealth from (2). The remaining terms capture stochastic transitions. For an unemployed worker, the last term is the expected gain from accepting an offer. For an employed worker, the last two terms are the expected gain from being poached by a better firm and the loss from exogenous separation.

Reservation wage. Since $W(a, w)$ is strictly increasing in w , an unemployed worker accepts an offer if and only if the offered wage is at least the reservation wage, $w \geq \underline{w}(a)$, where the reservation wage $\underline{w}(a)$ solves $W(a, \underline{w}(a)) = U(a)$. A worker employed at $w = b$ has the same flow income and faces the same search technology and offer distribution as an unemployed worker, and therefore $U(a) = W(a, b)$ for all a , which implies

$$\underline{w}(a) = b \quad \text{for all } a \geq \underline{a}. \quad (5)$$

This feature of the reservation wage follows [Lise \(2013\)](#) and rests on the assumption that unemployed and employed workers share the same search technology. Intuitively, the reservation wage is determined by the trade-off between the flow gain from accepting an offer and the option value of continued search. When unemployed and employed workers face identical search costs $\psi(s)$ and identical contact rates per unit of intensity λ , there is no option value of waiting and any offer with positive flow gain $w - b > 0$ is accepted, regardless of wealth.⁴

Consumption policy. Let $c_u(a)$ and $c_e(a, w)$ denote the consumption policies of unemployed and employed workers. The first-order conditions are

$$u'(c_u(a)) = U'(a), \quad (6)$$

$$u'(c_e(a, w)) = \partial_a W(a, w). \quad (7)$$

⁴In a model without search intensity, it is common to introduce a parameter that captures on-the-job search efficiency to generate a realistic degree of job-to-job transitions. This model has no counterpart to that, but the calibrated model nonetheless endogenously generates reasonable job-to-job transition rates, as I confirm in Section 4.

As discussed in [Achdou et al. \(2022\)](#), the borrowing constraint does not enter the HJB equations directly, and the first-order conditions hold with equality at $a = \underline{a}$. The requirement that saving must be nonnegative at the constraint yields the state-constraint boundary conditions:

$$U'(\underline{a}) \geq u'(r\underline{a} + b), \quad (8)$$

$$\partial_a W(\underline{a}, w) \geq u'(r\underline{a} + w). \quad (9)$$

Search policy. Let $s_u(a)$ and $s_e(a, w)$ denote the search-intensity policies. The first-order conditions are

$$\psi'(s_u(a)) = \lambda \int_{\underline{w}}^{\bar{w}} \max\{W(a, w) - U(a), 0\} dF(w), \quad (10)$$

$$\psi'(s_e(a, w)) = \lambda \int_{\underline{w}}^{\bar{w}} \max\{W(a, w') - W(a, w), 0\} dF(w'). \quad (11)$$

Workers equate the marginal disutility of search to the expected continuation gain from a contact. In [Appendix A.2](#), I establish—analytically where possible and numerically otherwise—several basic properties of the search policy functions. In particular, search intensity decreases with wealth for both unemployed and employed workers and, among employed workers, also decreases with the current wage. Moreover, the semi-elasticity of employed workers' search intensity with respect to the current wage is generally decreasing in wealth.

Stationary distributions. Let $G_u(a)$ denote the stationary distribution of unemployed workers over wealth, and let $G_e(a, w)$ denote the stationary joint distribution of employed workers over wealth and wages. Let $g_u(a)$ and $g_e(a, w)$ denote their densities on the interior of the asset domain. I write f for the density of the offer distribution F (which exists in equilibrium because the wage policy is strictly increasing, as shown below), $g_e(w) = \int_{\underline{a}}^{\infty} g_e(a, w) da$ for the wage marginal of the employed, and $g_e(a | w) = g_e(a, w)/g_e(w)$ for the conditional wealth density. Since accepted wages are bounded below by the reservation wage b , integrals over employed wages run from b . For $a > \underline{a}$, these densities satisfy the following

Kolmogorov forward (KF) equations:⁵

$$\begin{aligned} 0 = \partial_t g_u(a) = & -\partial_a \left((ra + b - c_u(a)) g_u(a) \right) - s_u(a) \lambda (1 - F(b)) g_u(a) \\ & + \delta \int_b^{\bar{w}} g_e(a, w) dw, \end{aligned} \quad (12)$$

$$\begin{aligned} 0 = \partial_t g_e(a, w) = & -\partial_a \left((ra + w - c_e(a, w)) g_e(a, w) \right) \\ & - \left[\delta + s_e(a, w) \lambda (1 - F(w)) \right] g_e(a, w) \\ & + s_u(a) \lambda f(w) \mathbb{1}\{w \geq b\} g_u(a) + \lambda f(w) \int_b^w s_e(a, x) g_e(a, x) dx, \end{aligned} \quad (13)$$

together with the adding-up condition

$$\int_{\underline{a}}^{\infty} g_u(a) da + \int_b^{\bar{w}} \int_{\underline{a}}^{\infty} g_e(a, w) da dw = 1. \quad (14)$$

In equation (12), the first term describes the reallocation of unemployed workers across asset levels, the second term is the outflow due to job finding, and the third term is the inflow from exogenous job destruction. In equation (13), the first term describes the reallocation of employed workers across asset levels. The second term is the outflow due to exogenous separation or job-to-job transitions, while the last two terms are inflows from unemployment and from lower-wage employment.⁶

Aggregate search intensity. Given the stationary distribution of workers, the aggregate search intensity is given by

$$S = \int_{\underline{a}}^{\infty} s_u(a) g_u(a) da + \int_b^{\bar{w}} \int_{\underline{a}}^{\infty} s_e(a, w) g_e(a, w) da dw. \quad (15)$$

⁵The full stationary distributions may contain a mass point at the borrowing constraint. For expositional convenience, expressions involving g_u and g_e in expectations and aggregates throughout the paper should be understood as shorthand for integration with respect to the full distributions G_u and G_e , including any boundary mass point. See Appendix E of [Achdou et al. \(2022\)](#) for a detailed discussion of measure-valued KF equations with boundary mass.

⁶As in [Achdou et al. \(2022\)](#), the KF equations are interpreted as equations for the stationary distribution induced by the optimal policies satisfying the state-constraint boundary conditions (8) and (9) at $\underline{a}$. Thus, no separate boundary condition is imposed on the KF equations at $\underline{a}$.

For notational convenience, I also define the aggregate search intensity of the unemployed, S_u , and the average search intensity of workers employed at wage w , $\bar{s}_e(w)$, as follows:

$$S_u = \int_{\underline{a}}^{\infty} s_u(a) g_u(a) da, \quad (16)$$

$$\bar{s}_e(w) = \int_{\underline{a}}^{\infty} s_e(a, w) g_e(a | w) da. \quad (17)$$

The object $\bar{s}_e(w)$ plays a central role in the firm problem. It summarizes the turnover risk a firm faces conditional on posting wage w , after integrating out worker wealth.

2.3 Firm Problem

The goal of this subsection is to derive the static maximization problem that is standard in the [Burdett and Mortensen \(1998\)](#) literature. In the limit $\rho_f \rightarrow 0$, firms maximize the steady-state flow profit

$$\max_{w, v} (y - w) \ell(w, v) - c(v), \quad (18)$$

where $\ell(w, v)$ is the stationary firm size. I derive the first-order conditions of (18) from a problem with worker wealth dynamics. The static formulation emerges as the $\rho_f \rightarrow 0$ limit of the first-order conditions, as is common in the literature.⁷

Throughout, an individual firm with productivity y takes as given the worker-side aggregates including the worker distributions g_u and g_e , the consumption policies c_u and c_e , the search policies s_u and s_e , and the contact rates λ and q . Since each firm is atomistic, firms also take the equilibrium wage-offer distribution F as given. All of these are pinned down in equilibrium. The aggregate environment is stationary throughout.

2.3.1 Match Value

Consider an ongoing match between a worker of wealth a and a firm of productivity y that has posted wage w . Let $J(a, w; y)$ denote the firm's present value of profits from this match, evaluated at the worker's current wealth a . The match generates flow profit $(y - w)$ as long

⁷[Burdett and Mortensen \(1998\)](#) explicitly take the limit $\rho/\lambda_0 \rightarrow 0$; see footnote 4 of their paper. [Bilal and Lhuillier \(2025\)](#) show that the limit of dynamic first-order conditions corresponds to the first-order conditions of the steady-state flow profit problem in a more general environment with decreasing returns to scale.

as it survives, and is destroyed at the wealth-dependent hazard rate

$$\tau(a, w) = \delta + s_e(a, w) \lambda (1 - F(w)), \quad (19)$$

where the first term is the exogenous job-destruction rate and the second is the endogenous job-to-job transition rate. While the match continues, the worker's wealth deterministically evolves according to $\dot{a} = ra + w - c_e(a, w)$, where $c_e(a, w)$ is the worker's consumption policy derived in Section 2.2.

Given these expressions, the firm's present discounted value of profits from the match is given by

$$J(a, w; y) = \int_0^\infty e^{-\rho_f t - \int_0^t \tau(a_k, w) dk} (y - w) dt, \quad (20)$$

where a_k denotes the wealth at time k after hiring along the path starting from $a_0 = a$ according to the law of motion $\dot{a}_k = ra_k + w - c_e(a_k, w)$. I rewrite this equation as

$$J(a, w; y) = (y - w) d(a, w; \rho_f), \quad (21)$$

where $d(a, w; \rho_f)$ is the expected *discounted* match duration and is given by

$$d(a, w; \rho_f) = \int_0^\infty \exp\left(-\rho_f t - \int_0^t \tau(a_k, w) dk\right) dt. \quad (22)$$

At $\rho_f = 0$, the expression is the expected match duration. The decomposition (21) factorizes the match value into per-period profit $(y - w)$ and discounted survival time $d(a, w; \rho_f)$, separating the firm-level profit margin from the worker-level wealth-related term.

2.3.2 Firm's Flow Problem

At each instant, the firm with productivity y chooses a posted wage w and vacancies v . A vacancy posted at wage w converts to a hire at rate

$$Q(w) = q \frac{S_u + \int_b^w \bar{s}_e(\tilde{w}) g_e(\tilde{w}) d\tilde{w}}{S}, \quad (23)$$

which I refer to as the vacancy yield. The vacancy yield is the contact rate times the probability of acceptance, which is the fraction of search intensity coming from workers either

unemployed or working at lower-wage firms. At posted wage w , the wealth distribution of newly hired workers is given by

$$h(a \mid w) = \frac{s_u(a) g_u(a) + \int_b^w s_e(a, \tilde{w}) g_e(a, \tilde{w}) d\tilde{w}}{S_u + \int_b^w \bar{s}_e(\tilde{w}) g_e(\tilde{w}) d\tilde{w}}. \quad (24)$$

Both the vacancy yield $Q(w)$ and the new-hire wealth distribution $h(a \mid w)$ are equilibrium objects that the individual firm takes as given. At each instant, the firm solves the following problem:

$$(w(y), v(y)) = \arg \max_{w, v} v Q(w) \mathbb{E}_h[J(a, w; y) \mid w] - c(v), \quad (25)$$

where the expectation over new-hire wealth is

$$\mathbb{E}_h[J(a, w; y) \mid w] = \int_a^\infty J(a, w; y) h(a \mid w) da. \quad (26)$$

The per-instant objective (25) depends on w and v but not on the firm's own workforce. In a stationary aggregate environment, the per-instant problem is therefore the same at every instant, and the firm's optimal posting schedule is constant in time. Constant returns to scale and aggregate stationarity together imply that the firm has no incentive to vary its posting across hiring cohorts, even though it could in principle do so.

Since the firm's optimization (25) requires only the population aggregates Q , h , F , $\tau(a, w)$ and the worker's policies, the within-firm wealth distribution does not enter the firm's first-order conditions. The firm's payoff-relevant state for choosing (w, v) is just y , the firm's own productivity. The within-firm wealth distribution is not a state the firm needs to track for its decisions.⁸

First-order conditions. The first-order conditions of (25) with respect to v and w are

$$c'(v) = Q(w) \mathbb{E}_h[J(a, w; y) \mid w], \quad (27)$$

$$Q'(w) \mathbb{E}_h[J(a, w; y) \mid w] + Q(w) \partial_w \mathbb{E}_h[J(a, w; y) \mid w] = 0. \quad (28)$$

⁸With decreasing returns to scale, the expected value of a new hire depends on the existing number of workers, and the wealth distribution of existing workers matters for the firm problem through their turnover rates.

The first equation equates the marginal cost of a vacancy to the marginal return, which is given by the hiring rate times the expected match value. The second equation describes how the firm chooses its posted wage: raising w increases $Q(w)$ (more applicants accept) but lowers $\mathbb{E}_h[J \mid w]$ (each hire is less profitable). The optimal wage balances the two margins.

The log of the wage objective has increasing differences in firm productivity and the posted wage (its cross-partial derivative with respect to (y, w) is $1/(y - w)^2 > 0$), which implies the optimal wage policy is weakly increasing in productivity. If wage choices are interior, the wage first-order condition further implies strict monotonicity.

With the policy functions $w(y)$ and $v(y)$, aggregate vacancies V and the wage-offer distribution $F(w)$ are given by

$$V = N \int_b^{\bar{y}} v(y) d\Gamma(y), \quad (29)$$

$$F(w) = \frac{N}{V} \int_b^{\bar{y}} \mathbb{1}\{w(y) \leq w\} v(y) d\Gamma(y). \quad (30)$$

2.3.3 The Static-Equivalent Problem and the $\rho_f \rightarrow 0$ Limit

I now connect the first-order conditions (27)–(28) to the static formulation (18). The key step is to express $\mathbb{E}_h[J(a, w; y) \mid w]$ in terms of the stationary firm-level turnover rate.

I define the new-hire expected discounted match duration as

$$D(w; \rho_f) = \mathbb{E}_h[d(a, w; \rho_f) \mid w] = \int_{\underline{a}}^{\infty} d(a, w; \rho_f) h(a \mid w) da. \quad (31)$$

Then the expected match value is given by $\mathbb{E}_h[J(a, w; y) \mid w] = (y - w) D(w; \rho_f)$, and the first-order conditions (27)–(28) become

$$c'(v) = (y - w) \tilde{n}(w; \rho_f), \quad (32)$$

$$\tilde{n}(w; \rho_f) = (y - w) \frac{\partial \tilde{n}(w; \rho_f)}{\partial w}, \quad (33)$$

where $\tilde{n}(w; \rho_f) = Q(w) D(w; \rho_f)$. The second first-order condition follows from substituting (21) into (28) and rearranging.

At general $\rho_f > 0$, the discounted-lifetime object $D(w; \rho_f)$ is determined by the wealth dynamics of new hires. Workers enter with wealth distribution $h(a \mid w)$, accumulate wealth

along their match, and separate at the wealth-dependent hazard $\tau(a, w)$. There is no closed-form expression for $D(w; \rho_f)$ in terms of cross-sectional objects. At $\rho_f = 0$, however, the expression greatly simplifies, as shown in the following lemma.

Lemma 2.1. *In a symmetric stationary equilibrium (defined in Section 2.4), $D(w; 0) = 1/T(w)$, where $T(w)$ is the firm-level turnover rate*

$$T(w) = \delta + \bar{s}_e(w) \lambda (1 - F(w)) \quad (34)$$

for w in the support of F .

Proof. See Appendix A.1. □

The lemma states that the expected match duration of a new hire equals the inverse of the *cross-sectional* firm-level turnover rate in stationary equilibrium. The equality is less obvious than it may appear. A new hire is not a random draw from the firm's existing workforce. Instead, entrants start from the new-hire wealth distribution $h(a | w)$, and their separation hazard $\tau(a, w)$ falls as they accumulate wealth over the match. The existing workforce, in turn, over-represents survivors with low hazards, and one might expect selection and wealth-driven duration dependence to make $D(w; 0)$ different from $1/T(w)$.

Stationarity nevertheless implies exact equality, and the logic requires three accounting facts about the pool of workers employed at wage w .⁹ First, by Little's law (Little, 1961), the stock equals the inflow rate times the average completed duration of entrants.¹⁰ Second, by flow balance, the inflow rate equals the outflow rate in a stationary pool. Third, by the definition of the turnover rate, the outflow rate equals $T(w)$ times the stock. Combining the three statements, the stock cancels, and the average duration of entrants equals $1/T(w)$, regardless of how individual hazards vary with wealth or tenure. With discounting, early and late separations receive different weights, and the cohort's wealth dynamics reappear in $D(w; \rho_f)$. At $\rho_f = 0$, the entire wealth-dependent cohort problem instead collapses into a single cross-sectional statistic, $T(w)$.

⁹Bradley and Gottfries (2021) use Little's law for the same purpose. The quit hazard in their model varies over the match, and the expected duration of a new hire nonetheless equals the inverse of the average leaving rate.

¹⁰Little's law establishes $L = \lambda W$ for a stationary queuing system, where, in Little's notation, L is the stock, λ is the arrival rate, and W is the *average* time spent in the system.

Given the firm-level turnover rate $T(w)$, I obtain the following first-order conditions under $\rho_f \rightarrow 0$:

Proposition 2.2 (Static equivalence at $\rho_f \rightarrow 0$). *As $\rho_f \rightarrow 0$, the first-order conditions (32)–(33) converge to*

$$c'(v) = (y - w) \tilde{n}(w), \quad (35)$$

$$\tilde{n}(w) = (y - w) \tilde{n}'(w), \quad (36)$$

where $\tilde{n}(w) = Q(w)/T(w)$ is the stationary firm size per vacancy, so that the stationary firm size in (18) is $\ell(w, v) = v \tilde{n}(w)$. These first-order conditions coincide with the ones obtained by maximizing the static steady-state flow profit problem (18) directly with respect to (w, v) .

The proposition formalizes the equivalence between (i) writing the match-level value (20), formulating the firm's per-instant flow problem (25), taking first-order conditions, and then taking $\rho_f \rightarrow 0$; and (ii) writing the static steady-state flow profit problem (18) directly and taking first-order conditions. Throughout the rest of the paper, I follow this convention and work with the limiting first-order conditions (35)–(36).

2.4 Equilibrium

A symmetric stationary equilibrium consists of: worker value functions U and W and policy functions c_u , c_e , s_u , and s_e ; worker densities g_u and g_e ; firm policy functions w and v ; wage-offer distribution F ; aggregate search intensity S , aggregate vacancies V , labor-market tightness θ , and contact rates λ and q such that:

1. *Worker optimality.* The value functions U, W and policies c_u, s_u, c_e, s_e jointly satisfy the HJB equations (3)–(4) subject to the borrowing constraint $a \geq \underline{a}$, the consumption first-order conditions (6)–(7), the search first-order conditions (10)–(11), and the reservation-wage condition $\underline{w}(a) = b$.
2. *Worker distributions.* g_u and g_e solve the KF equations (12)–(13) with adding-up (14).
3. *Firm optimality.* The wage and vacancy policies $w(y), v(y)$ solve the limiting first-order conditions (35)–(36) for each productivity $y \in [b, \bar{y}]$, where the firm-side schedules $Q(w), T(w), \tilde{n}(w)$ are defined in (23), (34), and Proposition 2.2.

4. *Wage-offer distribution.* F is generated by the firm wage policy via (30).
5. *Aggregate consistency.* Aggregate search intensity S is given by (15) and aggregate vacancies V by (29).
6. *Matching.* Labor-market tightness and contact rates satisfy $\theta = V/S$, $\lambda = \mathcal{M}(1, \theta)$, and $q = \mathcal{M}(\theta^{-1}, 1)$.

2.5 Discussion

Wage-posting protocol. The wage-setting protocol in this paper imposes three restrictions: firms post a single wage at a time, the wage is fixed over the duration of the match, and it is not renegotiated when outside offers arrive. A single wage reflects that a worker’s wealth is private information and not contractible. Firms may know the equilibrium wealth distribution of applicants at a given wage, but cannot price-discriminate on individual assets, and I study a pooling posted-wage benchmark. This restriction is in line with direct evidence of equal pay within establishments (Dube et al. 2019, Giupponi and Machin 2026).¹¹

Fixing the wage over the spell rules out tenure-contingent contracts as in Burdett and Coles (2003). Allowing them would add a retention instrument that partially substitutes for posted wages, shifting part of the wealth-search channel into within-firm wage growth without eliminating it.

No renegotiation rules out offer matching and sequential auctions (Postel-Vinay and Robin, 2002; Cahuc et al., 2006). Empirically, wages are typically not bargained in low-wage labor markets (Hall and Krueger 2012), and Di Addario et al. (2023) find that wages depend little on past employers, a pattern consistent with posted-wage models. According to Faberman et al. (2022), in the Survey of Consumer Expectations, counteroffers arise for about 12% of employed workers who receive an outside offer, a rate that, as they note, falls short of what sequential-auction models imply, although bargaining is not absent (38% of offers to employed workers involve some), and even rare counteroffers can matter through the threat of renegotiation.

¹¹Giupponi and Machin (2026) document that English care homes adopt uniform pay schedules within job roles, paying workers of different ages the same wage for the same job even when age-specific minimum wages permit differentiation. Dube et al. (2019) document that a large U.S. retailer adjusts pay through a uniform rule irrespective of individual characteristics and that worker quits respond sharply to deviations from peer pay. I believe that firms that do not tailor wages even to observable worker characteristics are unlikely to tailor them to the unobservable wealth of individual applicants.

Under these restrictions, the retention pressure generated by outside offers loads entirely onto posted wages and turnover, which makes the role of worker wealth in shaping the wage-offer distribution transparent. Accordingly, the results should be read as a benchmark for labor markets in which wages are posted and not systematically renegotiated.

Perfectly patient approximation. In practice, ρ_f matters only to the extent that it is large relative to $\tau(a, w)$; if it were, the match value would be dominated by discounting and dropping it would be a poor approximation. The discount rate, typically calibrated to about 0.05 annually, is, however, much smaller than annual turnover rates, which exceed 0.2. Appendix H confirms this numerically by solving the model without the approximation, discounting match profits at $\rho_f = \rho$ and pricing each match by equation (20): all calibration moments change by less than 5%, and most by less than 3%.

3 Firm Wage Setting

In this section, I investigate firms' wage-setting incentives and the role of worker-side wealth and job search in shaping them.

3.1 Wage Function

Recall that the wage function $w(y)$ solves the first-order condition:

$$\tilde{n}(w(y)) = (y - w(y)) \tilde{n}'(w(y)). \quad (36)$$

Define the firm size per vacancy as a function of productivity, $n(y) = \tilde{n}(w(y))$, and substitute it into the equation above; the wage policy then satisfies the first-order ODE $w'(y) = (y - w(y)) n'(y)/n(y)$. With the boundary condition $w(b) = b$, the wage function takes the following form:

$$w(y) = y - \int_b^y \frac{n(x)}{n(y)} dx. \quad (37)$$

This is the standard [Burdett and Mortensen \(1998\)](#) expression, with the difference that $n(y)$ here is endogenous to the wealth distribution and the search-intensity policies.¹² Also, this

¹²See Appendix A.3 for the derivation.

is not a complete characterization since n implicitly depends on the wage function through the job search policies of workers.

3.2 Decomposing Wage Markdown

The wage function (37) is useful but does not isolate the economic forces shaping wage setting at each point in the wage distribution. To isolate them, I rearrange the wage first-order condition as

$$\underbrace{y - w(y)}_{\text{markdown}} = \frac{1}{\epsilon_{\ell,w}(w(y))}, \quad (38)$$

where $\epsilon_{\ell,w}(w) = \frac{\partial \log \ell(w,v)}{\partial w}$ is the semi-elasticity of the firm size $\ell(w,v)$ with respect to the wage w , evaluated at $(w(y), v(y))$.¹³ If $\ell(w,v)$ is interpreted as a firm-level labor supply function, this expression equates the additive wage markdown to the reciprocal of the firm-level labor supply semi-elasticity. The labor supply semi-elasticity, in turn, decomposes into a poaching component and a retention component:

$$\epsilon_{\ell,w}(w) = \underbrace{\epsilon_{Q,w}(w)}_{\substack{\text{Poaching} \\ \text{semi-elasticity}}} + \underbrace{\epsilon_{1/T,w}(w)}_{\substack{\text{Retention} \\ \text{semi-elasticity}}}, \quad (39)$$

where $\epsilon_{Q,w}(w) = \frac{d \log Q(w)}{dw}$ is the poaching semi-elasticity and $\epsilon_{1/T,w}(w) = \frac{d \log 1/T(w)}{dw}$ is the retention semi-elasticity. A marginal increase in wages raises firm size both by drawing additional applicants (poaching margin) and by reducing the rate at which incumbents depart (retention margin). I can further decompose the retention margin. Let

$$\omega(w) = \frac{\bar{s}_e(w) \lambda (1 - F(w))}{\delta + \bar{s}_e(w) \lambda (1 - F(w))} \quad (40)$$

denote the share of turnover at posted wage w that comes from job-to-job separations. Then direct differentiation of $\log(1/T(w))$ with $T(w) = \delta + \bar{s}_e(w) \lambda (1 - F(w))$ gives

$$\epsilon_{1/T,w}(w) = \omega(w) \left[\underbrace{\frac{f(w)}{1 - F(w)}}_{\substack{\text{Upper-tail} \\ \text{hazard}}} + \underbrace{\epsilon_{\bar{s},w}(w)}_{\substack{\text{Employee search} \\ \text{semi-elasticity}}} \right], \quad (41)$$

¹³Since $\ell(w,v)$ is log separable between v and w , the derivative of its log with respect to w does not depend on v .

where $\epsilon_{\bar{s},w}(w) = -\frac{d \log \bar{s}_e(w)}{dw}$ is the semi-elasticity of the average search intensity $\bar{s}_e(w)$ of workers employed at wage w with respect to the wage.

Combining these expressions, the labor supply semi-elasticity can be decomposed into

$$\epsilon_{\ell,w}(w) = \epsilon_{Q,w}(w) + \omega(w) \left[\frac{f(w)}{1 - F(w)} + \epsilon_{\bar{s},w}(w) \right]. \quad (42)$$

Equation (42) expresses the labor supply semi-elasticity in terms of the four named, economically interpretable objects. I provide the interpretation of each component in order below.

The *poaching semi-elasticity* can be rewritten as

$$\epsilon_{Q,w}(w) = \frac{d \log Q(w)}{dw} = \frac{\bar{s}_e(w) g_e(w)}{S_u + \int_b^w \bar{s}_e(\tilde{w}) g_e(\tilde{w}) d\tilde{w}}.$$

The numerator is the density of marginal applicants, i.e., employees at wage w who could be poached upward, weighted by their search intensity. The denominator is the cumulative applicant pool below w . The poaching semi-elasticity captures how many additional workers the firm attracts by marginally raising its wage, relative to the pool of applicants it already attracts.

The *job-to-job share* $\omega(w)$ scales the retention margin. To the extent that turnover occurs through endogenous job-to-job transitions rather than exogenous destruction, the retention margin is amplified.

The *upper-tail hazard* $f(w)/(1 - F(w))$ is the standard [Burdett and Mortensen \(1998\)](#) local-competition object. It is the density of rival offers paying just above w relative to the mass above.¹⁴ Higher upper-tail hazards imply that slightly increasing a wage offer allows the firm to reduce the mass of offers that can poach its workers, and this puts upward pressure on wages.

The *employee search semi-elasticity* $\epsilon_{\bar{s},w}(w) = -d \log \bar{s}_e(w)/dw$ captures how strongly higher pay reduces search effort on average. It is the wage semi-elasticity of

$$\bar{s}_e(w) = \int s_e(a, w) g_e(a | w) da,$$

¹⁴Another way to see this is to note that $\frac{f(w)}{1-F(w)} \Delta$ is approximately the fraction of offers above w that lie in $[w, w + \Delta)$ for small $\Delta > 0$.

and its sign and magnitude reflect both (i) the within-worker response of $s_e(a, w)$ to w at each wealth level and (ii) the wealth composition of workers at wage w . Both channels depend on wealth. The policy $s_e(a, w)$ is decreasing in w because higher current wages lower the option value of search. In addition, its proportional response to the wage tends to be stronger for poorer workers, who face a higher marginal utility of consumption. I confirm this wealth gradient numerically in the model calibrated in Section 4.¹⁵

Role of wealth heterogeneity. The decomposition (42) clarifies where worker wealth heterogeneity enters firm wage setting. The poaching semi-elasticity and the employee search semi-elasticity are directly wealth-dependent. In contrast, the upper-tail hazard $f(w)/(1 - F(w))$ is not directly wealth-dependent; it depends only on the equilibrium wage-offer distribution. The job-to-job share $\omega(w)$, however, depends on the level of $\bar{s}_e(w)$ and is therefore wealth-dependent through cross-sectional employee search behavior. Crucially, the upper-tail hazard enters the markdown only multiplicatively through $\omega(w)$ in the retention block of (42), and wealth heterogeneity scales the strength with which between-firm competition from above translates into wage setting, even though wealth does not enter the upper-tail hazard directly.

The wealth dependence of the employee search semi-elasticity—its within-worker gradient and its compositional margin—is the sharpest object that distinguishes the present framework from a Burdett–Mortensen-style model without wealth heterogeneity. Later in the quantitative analysis, I demonstrate how this margin shapes the wage-setting behavior of firms.

4 Calibration

I solve the model at an annual frequency and use the implied transition rates to target labor market transition rates at an arbitrary frequency.

Quantitative extensions. When I calibrate the model to data, I slightly deviate from the model described in the previous sections in two respects, which I keep out of the main exposition to economize on notation. First, workers exit the labor market at rate μ and

¹⁵See Appendix A.2 for more details, including a shape restriction under which the gradient holds analytically.

are replaced by an equal measure of newborn workers who enter unemployment with zero wealth. I set $\mu = 1/40$, corresponding to a 40-year working life, and exits are not counted as EU transitions when computing labor market moments, consistent with the treatment of retirement in the data. Second, the job destruction rate depends on firm productivity, $\delta(y)$, declining log-linearly in y as specified below. All equilibrium conditions carry over with δ replaced by $\delta(y)$, with one exception: a firm contemplating a wage deviation retains its own destruction rate since $\delta(y)$ does not directly depend on the wage, and the dependence of turnover on $\delta(y)$ adds a correction term to the wage first-order condition. Setting $\mu = 0$ and a flat profile $\delta(y) = \delta$ recovers exactly the model of the previous sections. Appendix B provides the full model with both extensions.

4.1 Functional Forms

The utility function takes the form of constant relative risk aversion (CRRA) with the risk aversion parameter σ : $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. I slightly modify the search cost function assumed in Section 2.1.1 and set $\psi(s) = \psi_0 \frac{(s-\underline{s})^{1+\eta_s}}{1+\eta_s}$, where $\underline{s}$ is a model parameter representing the minimum search intensity a worker can choose without any cost, motivated by empirical evidence in Faberman et al. (2022) that workers sometimes receive unsolicited job offers without sending applications.¹⁶ The vacancy cost function takes the iso-elastic form of $c(v) = c_0 \frac{v^{1+\eta_v}}{1+\eta_v}$. The matching function is given by a Cobb–Douglas function $\mathcal{M}(S, V) = AS^{\eta_m} V^{1-\eta_m}$. The exogenous productivity distribution follows a four-parameter Beta distribution $\text{Beta}(\alpha, \beta)$ with support $[\underline{y}, \bar{y}]$.

4.2 Externally Set Parameters

I first externally set or normalize some parameters. I set the interest rate to $r = 0.03$ following Lise (2013). The discount rate ρ is calibrated internally to match household liquid wealth holdings, as described later. I set the borrowing constraint to $\underline{a} = 0$. I set the risk aversion parameter to 2.0, which is standard in the literature. I set the income of unemployed workers to $b = 0.4$, and given this normalization, I later calibrate the productivity distribution. In equilibrium, firms with productivity less than the reservation wage do not post vacancies. Since the reservation wage is b , I set the lower end of the support to $\underline{y} = b$. I set the maximum

¹⁶Bagger and Lentz (2019) also assume the same functional form.

productivity to $\bar{y} = 2.0$, five times the lowest productivity. I set the elasticity parameter of the matching function to $\eta_m = 0.5$ following [Petrongolo and Pissarides \(2001\)](#). I normalize the scale of the search cost function and the vacancy cost function at $\psi_0 = c_0 = 1$ since they cannot be separately identified from the matching efficiency.

Directly inferred parameters. The average firm size in the model is $\frac{1-u}{N}$, which identifies the measure of firms N given the unemployment rate u . Later I target $u = 0.06$, which implies $N = \frac{1-0.06}{\text{avg. firm size}}$. The average firm size of 23.3 reported in [Bilal et al. \(2022\)](#) leads to $N = 0.04$.

4.3 Parameters Estimated Internally

The remaining nine parameters $(A, \eta_s, \eta_v, \underline{s}, \alpha, \beta, \rho, \bar{\delta}, \zeta_\delta)$ are chosen to fit model moments to data counterparts. Although all the data moments jointly pin down all the estimated parameters, I briefly discuss which information in the data is particularly informative about which parameter.

Discount rate. With the interest rate fixed, the discount rate ρ governs how much wealth workers accumulate to self-insure against job loss. I calibrate ρ to match a median liquid wealth of workers equal to 4.35 months of income, following [Chaumont and Shi \(2022\)](#), who target the corresponding moment in the 2007 Survey of Consumer Finances (a ratio of median monthly income to liquid wealth of 23 percent).

Job destruction. The job destruction rate declines with firm productivity according to the log-linear profile $\delta(y) = \bar{\delta} \exp(-\zeta_\delta(y - y_{\text{mid}}))$, where y_{mid} is the midpoint of the productivity support, following [Birinci and See \(2023\)](#). The level $\bar{\delta}$ is disciplined by the aggregate monthly EU transition rate of 1.5 percent. The slope ζ_δ is disciplined by the ratio of the EU rate in the bottom quintile of the labor earnings distribution to that in the top quintile, which is 5.29 in the SIPP ([Birinci and See, 2023](#)).

Search cost function. [Faberman et al. \(2022\)](#) use directly measured search intensity of workers in the Survey of Consumer Expectations (SCE) to estimate the elasticity of employed workers' search intensity with respect to residualized current wages, which I denote by $\varepsilon_{s,w}^F$.

The unit of search intensity in Faberman et al. (2022) is either the number of job applications or the hours of job search activity. They then use it to discipline their search cost function curvature in their model. I similarly use their reduced-form elasticity $\varepsilon_{s,w}^F$ to discipline the curvature of search costs η_s in my model.

The unit of search intensity in my model is not the same as in Faberman et al. (2022). Appendix F nonetheless shows that the elasticity $\varepsilon_{s,w}^F$, which is a unit-free object, is informative about the curvature η_s of the search cost function in my model. The free-search level $\underline{s}$ is separately disciplined by the monthly rate of unsolicited job offers of 2.6% in the SCE (Faberman et al., 2022).

Vacancy cost curvature. The curvature of the vacancy cost function η_v is informed by the employment share of large firms. To see this, note that given the normalization $c_0 = 1$, the first-order condition for vacancy creation is

$$v(y) = \left((y - w(y)) \tilde{n}(w(y)) \right)^{\frac{1}{\eta_v}}. \quad (43)$$

Letting $\ell(y) = v(y) \tilde{n}(w(y))$ denote firm size, I can substitute $\tilde{n} = \ell/v$ into this condition and rearrange to obtain

$$\log\left(\frac{v(y)}{\ell(y)}\right) = \frac{1}{1 + \eta_v} \log(y - w(y)) - \frac{\eta_v}{1 + \eta_v} \log \ell(y). \quad (44)$$

Holding the profit flow $y - w(y)$ fixed, the vacancy rate v/ℓ declines with firm size with elasticity $-\eta_v/(1 + \eta_v)$, which vanishes as $\eta_v \rightarrow 0$ and approaches -1 as $\eta_v \rightarrow \infty$. Hence, the larger η_v is, the fewer vacancies large firms post relative to their size, and the smaller the employment share of large firms. I target the employment share of firms with 500 or more workers, which is 0.52 in the Census Business Dynamics Statistics according to Bilal et al. (2022).

Remaining parameters. The matching efficiency A directly affects the job finding rate and hence monotonically affects the steady-state unemployment rate. I pin it down by targeting the steady-state unemployment rate of 6%.

The shape parameters (α, β) jointly determine the shape of the productivity distribution,

Table 1: Model Parameters

Parameters	Description	Source/Target	Value	Model moment	Data moment
Panel A: Directly calibrated parameters					
r	Interest rate	Lise (2013)	0.03		
$\underline{a}$	Borrowing limit		0.00		
σ	Relative risk aversion		2.0		
b	Unemployment income	Normalization	0.4		
$\bar{y}$	Maximum productivity		2.00		
η_m	Matching function elasticity	Petrongolo and Pissarides (2001)	0.5		
μ	Labor market exit rate	40-year working life	0.025		
ψ_0	Job search cost: scale	Normalization	1.0		
c_0	Vacancy cost: scale	Normalization	1.0		
Panel B: Directly inferred parameters					
N	Measure of firms	Avg. firm size of 23.3	0.04		
Panel C: Estimated parameters					
η_s	Job search cost: curvature	Search-wage elasticity	1.89	-0.36	-0.36
η_v	Vacancy cost: curvature	Empl. share of firms sized 500+	2.07	0.52	0.52
A	Matching efficiency	Unemployment rate	9.97	0.06	0.06
$\underline{s}$	Free search	Unsolicited offer (monthly)	0.11	0.026	0.026
α	Productivity distribution	UI replacement rate	2.33	0.40	0.40
β	Productivity distribution	Std. dev. firm wage effects	10.00	0.20	0.20
ρ	Discount rate	Median liquid wealth (months of income)	0.052	4.35	4.35
$\bar{\delta}$	Job destruction: level	EU rate (monthly)	0.14	0.015	0.015
ζ_δ	Job destruction: slope	EU rate Q1/Q5, earnings dist.	3.00	5.29	5.29

Note: Panel A reports the directly calibrated parameter values. Panel B reports the parameter value that is directly inferred from data. Panel C reports the parameter values that are chosen to match model moments and data moments.

and they have distinct effects on the mean and the variance of productivity y .¹⁷ With b fixed, I target the average replacement rate $b/\mathbb{E}[w]$ of 0.4 so that unemployment income equals 40% of the average wage, which is informative about the mean wage and hence the mean productivity. I also target the firm-effect component of the log wage dispersion in the data, which is informative about the dispersion of productivity. [Kline \(2024\)](#) reports that bias-corrected standard deviations of firm fixed effects take values around 0.2, which I use as a target.¹⁸

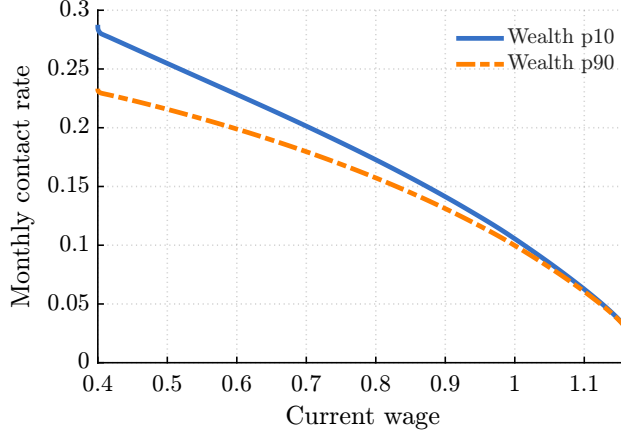
4.4 Estimation Results

Parameter values. Table 1 summarizes the model parameters and the targeted moments. In Appendix G, I support the heuristic arguments of the previous subsection by computing

¹⁷The mean and the variance of the Beta distribution over $[0, 1]$ are given by $\frac{\alpha}{\alpha+\beta}$ and $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

¹⁸See Figure 3 of [Kline \(2024\)](#).

Figure 1: Search Intensity by Wealth



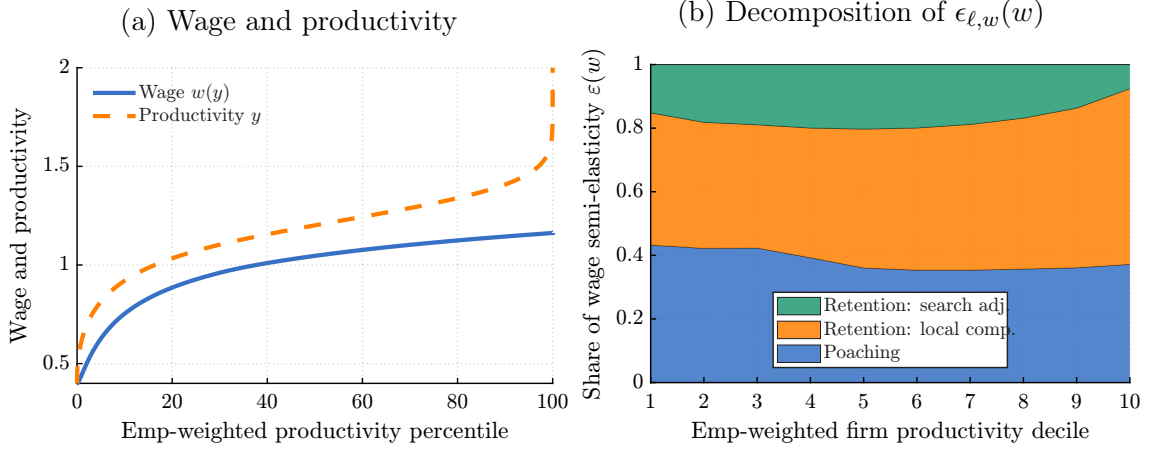
Note: The search policy $s_e(a, w)$ evaluated at the 10th and 90th percentiles of the wealth distribution. Values of $s_e(a, w)$ are multiplied by $\lambda/12$ to represent monthly contact rates.

how each targeted moment responds to a small perturbation in each parameter. I find that the moment associated with each parameter is indeed strongly informative about that parameter relative to other moments.

Search behavior of rich and poor workers. Figure 1 displays the search policy function $s_e(a, w)$ evaluated at $a = a_{p10}$ and $a = a_{p90}$, where a_{p10} and a_{p90} denote the 10th and 90th percentiles of the wealth distribution, respectively. The search intensity $s_e(a, w)$ is multiplied by $\lambda/12$ so that it represents the monthly contact rate. At the bottom of the job ladder, poor workers search much more intensively than wealthy workers in order to climb the ladder quickly and accumulate wealth, while this difference gradually diminishes as wages rise. The contact rate remains positive even at the top of the ladder since workers continue to receive unsolicited offers.

Wage schedule and the anatomy of markdowns. Figure 2 displays the estimated wage schedule and interprets it through the decomposition of the wage semi-elasticity derived in Section 3.2. Panel (a) plots the wage schedule and productivity against the employment-weighted productivity percentile, so that the horizontal axis reads as the position of workers in the employment distribution. Wages track productivity closely at the bottom of the ladder, where competition for workers is intense, and the markdown $y - w(y)$ widens toward the top, where firms face few poaching threats. Panel (b) decomposes the labor supply

Figure 2: Wage Schedule and the Decomposition of the Semi-Elasticity



Note: Panel (a) plots the wage schedule $w(y)$ and productivity y against the employment-weighted productivity percentile. The vertical gap is the markdown $y - w(y)$, which equals the inverse semi-elasticity $1/\epsilon_{\ell,w}(w)$. Panel (b) decomposes $\epsilon_{\ell,w}(w)$ into the poaching component $\epsilon_{Q,w}(w)$, the local-competition component of retention $\omega(w) \frac{f(w)}{1-F(w)}$, and the search adjustment $\omega(w)\epsilon_{\bar{s},w}(w)$, following equation (42), by employment-weighted productivity decile.

Table 2: Productivity Dispersion and Wage-to-Productivity Ratio

Moment	Model	Data	Source
Std. dev. log productivity	0.24	0.31-0.36	Blackwood et al. (2021)
Avg. wage-to-productivity ratio	0.84	0.74-0.91	Seegmiller (2025)

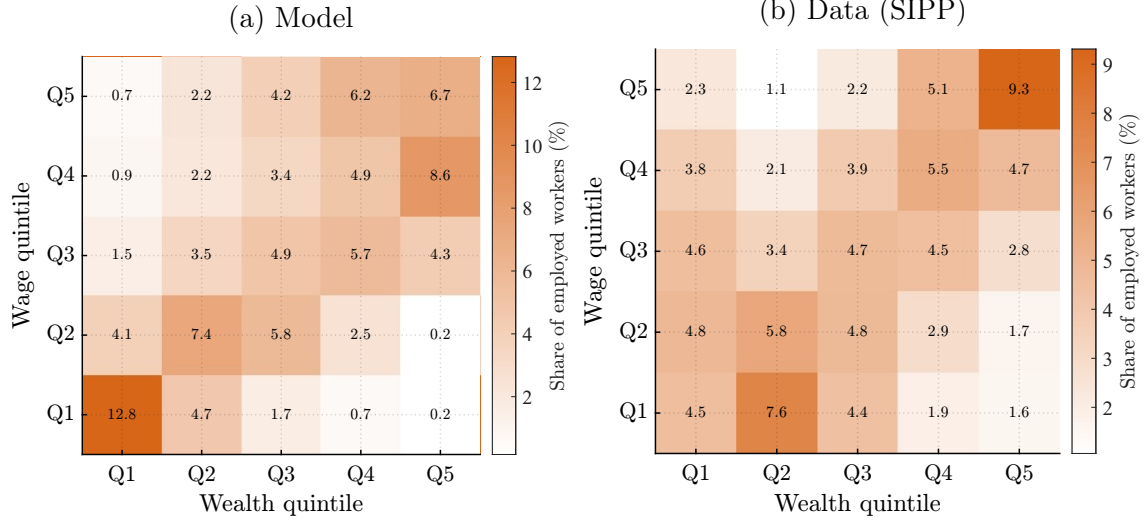
Note: This table reports firm-side non-targeted moments in the model and the data. The standard deviation of log firm productivity in the data comes from [Blackwood et al. \(2021\)](#) Table 1. The wage-to-productivity ratio in the model is defined as $\frac{w(y)}{y}$ for each y , and the average is taken across firms. The data counterpart is the average ratio of wages to marginal products (the markdown ratio) reported in [Seegmiller \(2025\)](#) Table 6.

semi-elasticity $\epsilon_{\ell,w}(w)$ into its three components in equation (42). The poaching component $\epsilon_{Q,w}(w)$ accounts for 35–43% of the semi-elasticity, and the local-competition component of retention $\omega(w) \frac{f(w)}{1-F(w)}$ accounts for 39–55%, rising toward the top of the ladder. The search adjustment $\omega(w)\epsilon_{\bar{s},w}(w)$ accounts for the remaining 8–20%, largest in the middle of the ladder and smallest at the very top.

4.5 Model Validation

Productivity dispersion and markdown. As a validity check on the firm side, I compare non-targeted model moments with their data counterparts, as reported in Table 2.

Figure 3: Joint Distribution of Wealth and Wages



Note: Each panel reports the share of employed workers (in percent) in each wealth-quintile $\times$ wage-quintile cell. Panel (a) is computed from the stationary joint distribution of employed workers over wealth and wages in the calibrated model. Panel (b) is computed from the SIPP sample described in Appendix D, where wealth is net liquid wealth and the wage is proxied by monthly earnings. Quintiles are computed from the respective marginal distributions, using person weights in the data, so that each row and each column sums to 20%. Under independence between wealth and wages, every cell would contain 4%.

The estimated productivity distribution implies a standard deviation of log firm productivity of 0.24. In the model, productivity dispersion is disciplined only through firm-driven wage dispersion in the data, and there is no guarantee that the estimated productivity dispersion matches the empirical one. Nevertheless, it is only modestly below the plant-level log-productivity standard deviation of 0.31–0.36 estimated by [Blackwood et al. \(2021\)](#) using U.S. data.

Firms in the model face heterogeneous competitive pressure, resulting in different wage markdowns across the productivity distribution. The estimated model implies an average wage-to-productivity ratio of 0.84, meaning that firms retain 16 cents per dollar of output. [Seegmiller \(2025\)](#) reports a corresponding ratio of wages to marginal products of 0.74–0.91 using U.S. employer-employee matched data. The model’s 0.84 falls in the middle of these estimates.

Joint distribution of wealth and wages. The interaction between wealth accumulation and the position on the job ladder is at the core of the model. Figure 3 compares the joint distribution of employed workers across wealth and wage quintiles in the model with its

Table 3: Flow Rates by Wealth Quintile Relative to the Aggregate

	Data (SIPP)					Model				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
$E \rightarrow U$	1.82	1.15	0.87	0.70	0.54	1.76	1.10	0.86	0.69	0.59
$U \rightarrow E$	0.88	1.08	1.04	1.10	1.06	1.10	1.07	1.00	0.94	0.89
$E \rightarrow E$	0.95	0.99	1.01	1.02	1.02	0.99	1.00	1.00	1.00	1.01
$U \rightarrow U$	1.07	0.97	0.97	0.95	0.91	0.97	0.98	1.00	1.02	1.03
$J \rightarrow J$	1.29	1.12	0.94	0.88	0.79	2.90	1.09	0.59	0.30	0.12

Note: Flow rates by wealth quintile relative to the aggregate flow rate. The data columns report the SIPP estimates of [Krusell et al. \(2017\)](#), Table 6, where wealth is household net worth. Model rates are computed from the stationary distribution; $E \rightarrow E$ counts all continuing employment and $J \rightarrow J$ job-to-job transitions, following [Krusell et al. \(2017\)](#).

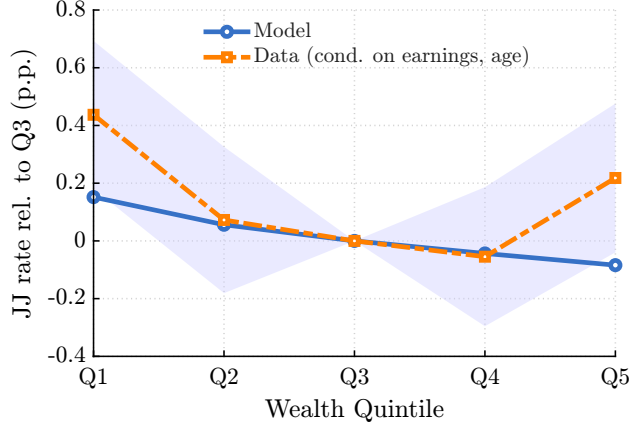
empirical counterpart constructed from the SIPP.

The model reproduces the positive wealth-wage sorting observed in the data. In both panels, the mass of workers concentrates along the diagonal: the top-right cell contains 6.7% of employed workers in the model and 9.3% in the data, well above the 4% benchmark under independence, and the off-diagonal corners are nearly empty. The model thus generates a realistic degree of cross-sectional dependence between wages and wealth even though no joint moment of the two is used in the calibration. The rank correlation between wealth and wage quantiles is 0.61 in the model and 0.37 in the data. The model overstates the positive association, which is somewhat expected: the job ladder is the only source of heterogeneity in the model, and therefore wealth is closely tied to the current rung of the ladder, whereas other unmodeled heterogeneity could weaken the association in the data.

Aggregate job-to-job transitions. The model also implies a monthly job-to-job transition rate of 2.1%, consistent with the roughly 2-3% in aggregate U.S. data ([Fallick and Fleischman, 2004](#); [Fujita et al., 2024](#)). This rate is not directly targeted in the estimation, serving as another overidentification check on the search technology.

Labor market flows by wealth. [Krusell et al. \(2017\)](#) report labor market flow rates by wealth quintile in the SIPP, and Table 3 compares the model with their estimates row by row. Although only the EU gradient across *earnings* quintiles is targeted, the model reproduces

Figure 4: Job-to-Job Transitions and Wealth Conditional on Wages



Note: Monthly job-to-job transition rates by wealth quintile, relative to the middle quintile, conditional on earnings and age. The data series reports coefficients from a linear regression of an indicator for a job-to-job transition on wealth quintile dummies, controlling for a quadratic in age and for earnings-decile, year, month, state, education, and interview-seam fixed effects, for male employed workers aged 25–55 in the private sector in the 1996–2008 SIPP panels; wealth is net liquid wealth excluding retirement accounts. Shaded regions report 95% confidence intervals. The model series fixes the wage composition at the aggregate employment distribution, the model counterpart of the earnings controls.

the EU gradient across wealth quintiles very closely. The UE gradient is downward-sloping in the model, reflecting the negative effect of wealth on job search, while it is slightly positive in the data. The EE (continuing employment) row and UU (continuing unemployment) row are approximately flat in both the model and the data. The JJ (job-to-job transition) row is steeper in the model than in the data, reflecting the composition effect previously described. With a single dimension of heterogeneity, wealth is closely tied to the rung of the ladder, and the raw gradient overstates the conditional one. In the next paragraph, I look into the relationship conditional on the rung of the ladder.

Job-to-job transitions and wealth conditional on wage. Since wealth and the position on the job ladder are correlated, the raw gradient of job-to-job mobility in wealth mixes the direct effect of wealth on search with the composition of ladder positions across wealth levels. Figure 4 isolates the direct channel. It plots job-to-job transition rates by wealth quintile conditional on earnings. In the data, workers in the bottom wealth quintile make distinctly more job-to-job transitions than equally paid but wealthier workers: the bottom-quintile coefficient of 0.44 p.p. per month is highly significant. Above the bottom quintile the conditional gradient is roughly flat. The model reproduces this asymmetric pattern,

Table 4: Median Saving Rates by Income Quintile (Percent)

	Q1	Q2	Q3	Q4	Q5
Data (PSID, active saving)	1.0	3.3	6.3	6.7	13.7
Model	0.0	4.0	3.9	2.9	1.3

Note: Median saving rates as a percent of disposable income by quintile of current income. The data row reports the PSID active-saving estimates of [Dynan et al. \(2004\)](#), Table 3, which exclude capital gains and pension saving, the two saving concepts absent from the model. The model row reports median saving rates of employed workers by wage quintile.

albeit weakly. The marginal value of climbing the job ladder is high when the borrowing constraint binds, and therefore the wealth-driven search response is concentrated among the poorest workers. It fades quickly once workers hold even a modest buffer stock.

Saving rates. Wealth accumulation in the model reflects saving out of income, and there are no capital gains and no pension saving. The natural data counterpart is the PSID *active saving* measure of [Dynan et al. \(2004\)](#)—the change in wealth net of capital gains—further excluding pension contributions, so that both saving concepts absent from the model are stripped out. Table 4 compares median saving rates by wage quintile in the model with their estimates. The model matches the first two quintiles reasonably well but fails to reproduce the strong increase in saving at higher incomes, where saving in the data might partly reflect life-cycle behavior and bequest motives that the model abstracts from.

Direct evidence on search activity. Job-to-job transitions measure realized mobility, which reflects search intensity only jointly with offer arrival and acceptance. Appendix E complements this evidence with a direct measure of search using the National Longitudinal Survey of Youth 1997 (NLSY97). The NLSY97 asks currently employed workers whether they have done anything to look for work in the past three months. Conditional on the wage and demographics, workers in the bottom liquid-wealth quintile are about a third more likely to report on-the-job search than workers in the top quintile, highlighting the influence of wealth on job search.

5 The Wealth Origins of Firm Wage Setting

In the calibrated model, workers' wealth shapes firm wage setting through search behavior: the wealth distribution along the job ladder determines both how intensely employees search and how sensitively their search responds to wages, and thereby the labor supply elasticities that discipline firms. This section quantifies that link by asking how much of the equilibrium wage structure originates from workers' wealth. Answering this question requires an experiment that removes the role of wealth from the economy while changing nothing else. I implement it by considering the economy where workers are perfectly insured. I also solve the opposite extreme case where workers have no access to saving at all so that the baseline economy can be placed on the full spectrum between zero and complete insurance against job-loss risk.

5.1 A Perfect-Insurance Benchmark

I construct a perfect-insurance benchmark by giving workers perfect insurance within a representative family, following the formulation of [Merz \(1995\)](#) and [Andolfatto \(1996\)](#).¹⁹ Each member is indexed by i , and all workers pool their income in a family that chooses a common consumption level C and each member's search intensity s_i to maximize the sum of members' utilities,

$$\max \int_0^\infty e^{-\rho t} \left[u(C_t) - \int \psi(s_{it}) di \right] dt \quad \text{s.t.} \quad \dot{A}_t = rA_t + \int inc_{it} di - C_t, \quad (45)$$

where inc_{it} is member i 's income (w if employed at wage w , and b if unemployed) and A_t denotes family assets subject to the same borrowing constraint as in the baseline economy. Since consumption is optimally equalized across members, an individual member's wealth and employment history are irrelevant to her consumption. In other words, the precautionary motive that drives behavior in the baseline economy is absent by construction.

Member i 's value to the family is the expected present value of the income stream she contributes to the common pool. Denoting by $V(w)$ the value of a member employed at wage w and by V_u the value of an unemployed member, both expressed in units of income,

¹⁹An alternative approach would be to impose risk neutrality. Although this might appear straightforward, it requires recalibration since the search cost function $\psi(s)$ in the baseline is measured in utility units. The representative-family benchmark allows me to cleanly transform the economy into one with effectively risk-neutral workers.

these values satisfy

$$\rho V(w) = \max_s \left[w - \frac{\psi(s)}{u'(C)} + s\lambda \int_w^{\bar{w}} (V(w') - V(w)) dF(w') + \delta(V_u - V(w)) \right], \quad (46)$$

$$\rho V_u = \max_s \left[b - \frac{\psi(s)}{u'(C)} + s\lambda \int_b^{\bar{w}} (V(w') - V_u) dF(w') \right]. \quad (47)$$

Appendix A.4 derives (46)–(47) from the family’s planning problem as envelope conditions for the shadow values of members. As elsewhere in the main text, the displays omit the quantitative extensions of Section 4.²⁰ Since $\rho > r$ in the calibration, the family holds no assets in the steady state, and consumption equals aggregate labor income: $C = bu + \int w g_e(w) dw$, where u is the unemployment rate and $g_e(w)$ is the density of employed workers over wages. The firm problem, the matching process, and the definition of equilibrium are unchanged. The benchmark removes only the wealth-dependence of search while keeping the search margin itself active. Appendix I reports the complementary experiment that fixes search effort at a constant level, separating the role of wealth-driven search heterogeneity from that of endogenous search effort itself.

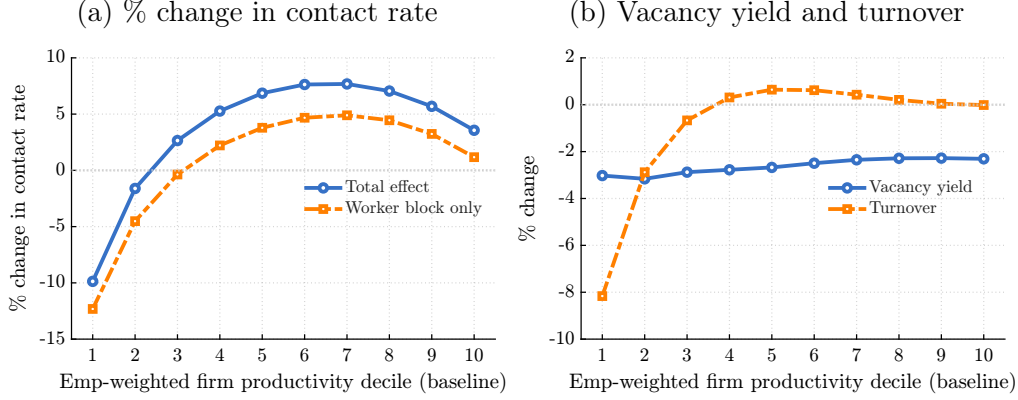
5.2 Search Behavior with Perfect Insurance

Panel (a) of Figure 5 shows how on-the-job search changes when the role of wealth is removed. The response is strongly non-monotonic: search intensity falls by roughly 10% at the bottom decile of the job ladder and rises by 5 to 8% through the middle, fading toward the top. The dashed line shows that this pattern is not an equilibrium feedback: it emerges already when the family’s search decisions are evaluated at the baseline firm policies and contact rates. Equilibrium adjustment then shifts the whole profile upward.

The pattern reveals how wealth shapes search in the baseline economy. Wealth accumulation due to incomplete markets tilts search effort toward the bottom of the job ladder. Workers at the lowest rungs are disproportionately wealth-poor, and their precautionary motive pushes their search intensity above what is implied by the pure income gain from climbing the ladder. In contrast, workers in the middle of the ladder have accumulated buffer

²⁰In the computations, the destruction rate is the productivity-dependent $\delta(y)$ of Section 4, expressed as a function of the wage through the inverse of the wage policy $w(y)$, and member values are additionally discounted at the exit rate μ , with exiting members replaced by newborn unemployed members as in the baseline demographics. See Appendix B for the full model.

Figure 5: Search and Turnover with Perfect Insurance



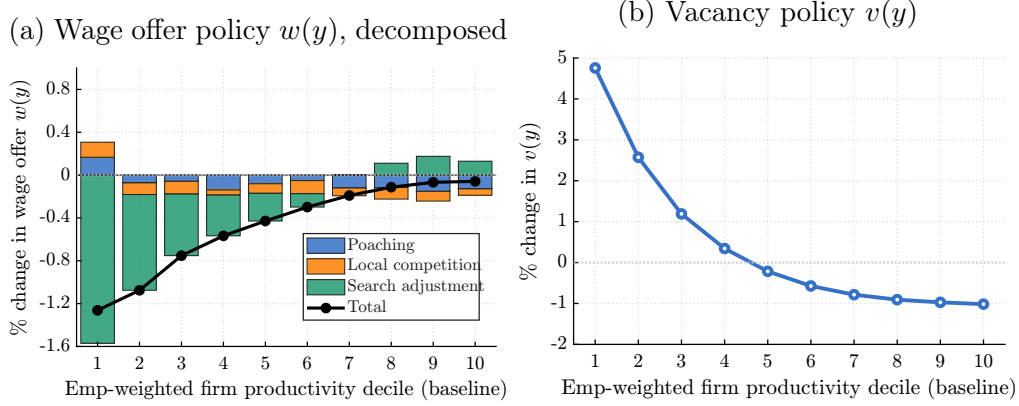
Note: Panel (a) displays the percentage change in the average monthly contact rate of employees $\lambda \bar{s}_e(w(y))/12$ from the baseline economy to the perfect-insurance benchmark, by baseline employment-weighted firm productivity decile. The dashed line fixes the firm-side solutions and the aggregate contact rates at their baseline values. Panel (b) displays the percentage change in the vacancy yield $Q(w(y))$ and the turnover rate $T(w(y))$, by the same deciles.

stocks that dampen their search incentives. Perfect insurance flattens the search profile. The heightened search at the bottom disappears, and the wealth-dampened search in the middle recovers. Panel (b) shows the resulting changes in the vacancy yield and turnover faced by firms, which transmit the search response to wage setting.

5.3 Wage Setting with Perfect Insurance

Panel (a) of Figure 6 shows the response of wage offers. Wage offers fall throughout the productivity distribution, with the decline concentrated at the lower deciles. Firms in the bottom decile cut wages by 1.3%, while offers above the median fall by less than half a percent. The stacked bars decompose the change into the contributions of the three components of the labor supply semi-elasticity in equation (42), and the decomposition is exact. Let k index the three components, and let $\sigma_k(y)$ denote the share of component k in the semi-elasticity $\epsilon_{\ell,w}(w(y))$, so that $\sum_k \sigma_k(y) = 1$ in each economy. Specifically, let $\sigma_1(y) = \frac{\epsilon_{Q,w}(w(y))}{\epsilon_{\ell,w}(w(y))}$, $\sigma_2(y) = \frac{\omega(w(y)) \frac{f(w(y))}{1-F(w(y))}}{\epsilon_{\ell,w}(w(y))}$, $\sigma_3(y) = \frac{\omega(w(y)) \epsilon_{\bar{s},w}(w(y))}{\epsilon_{\ell,w}(w(y))}$. By the markdown equation (38), the wedge $y - w(y)$ equals the inverse of the semi-elasticity, and the shares describe its composition. With the baseline and the perfect-insurance economies denoted by super-

Figure 6: Firm-Side Responses to Perfect Insurance



Note: Panel (a) displays the percentage change in the wage-offer policy $w(y)$ from the baseline economy to the perfect-insurance benchmark by baseline employment-weighted firm productivity decile (solid line), decomposed into the contributions of the three components of the labor supply semi-elasticity in equation (42): poaching $\epsilon_{Q,w}(w)$, local competition $\omega(w) \frac{f(w)}{1-F(w)}$, and search adjustment $\omega(w) \epsilon_{\bar{s},w}$ (stacked bars), following the exact decomposition in equation (48). The three contributions sum to the total change. Panel (b) displays the percentage change in the vacancy policy $v(y)$.

scripts Base and PI, the change in the wage offer splits as

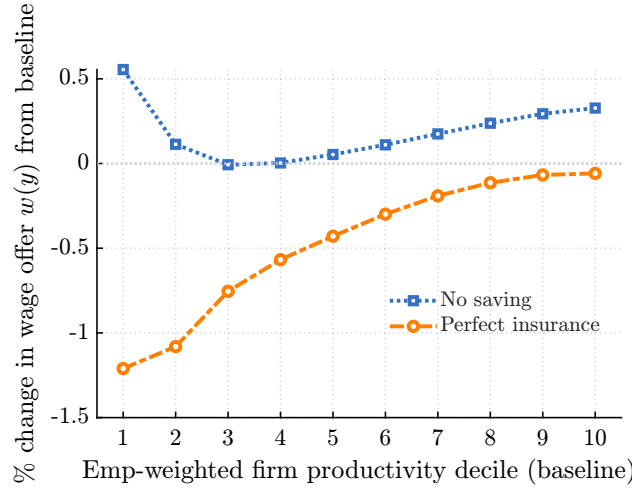
$$w^{\text{PI}}(y) - w^{\text{Base}}(y) = \sum_k \underbrace{\left[\sigma_k^{\text{PI}}(y) (y - w^{\text{Base}}(y)) - \sigma_k^{\text{Base}}(y) (y - w^{\text{PI}}(y)) \right]}_{\text{contribution of component } k}, \quad (48)$$

which holds as an identity because the shares sum to one in each economy. Panel (a) plots each contribution relative to the baseline wage $w^{\text{Base}}(y)$, averaged within the baseline employment-weighted deciles. If a component's share were unchanged between the two economies, its bar would simply be that share of the total change.

The decomposition shows that the wage cuts at the bottom are driven almost entirely by the *search-adjustment channel* $\epsilon_{\bar{s},w}(w)$ scaled by $\omega(w)$: in the baseline, the intense and wage-sensitive search of wealth-poor workers at the bottom rungs makes the retention margin steep for low-productivity firms, forcing them to pay wages well above what the other competition channels alone would sustain. With perfect insurance, this discipline disappears and firms reduce wage offers substantially. In the upper half of the ladder the sign flips. Recovered search intensity strengthens the search-adjustment channel, offsetting weaker poaching and local competition, so that wages there barely move.

Panel (b) shows the accompanying vacancy response. Low-productivity firms, facing

Figure 7: Wage Offers at the Two Poles of the Insurance Spectrum



Note: This figure displays the percentage change in the wage-offer policy $w(y)$ from the baseline economy to the no-saving (hand-to-mouth) economy and to the perfect-insurance benchmark, by baseline employment-weighted firm productivity decile.

cheaper labor and higher vacancy yields, expand vacancy postings by up to 5% at the bottom of the ladder, while firms above the fourth decile reduce postings slightly. The expansion of low-wage job creation shifts the composition of offers down the ladder.

5.4 Aggregate Impact

To place the baseline economy on the full spectrum between zero and complete insurance, I also solve the opposite extreme, in which workers consume their income each instant ($c = w$ if employed, $c = b$ if unemployed). In this hand-to-mouth economy, wealth heterogeneity is again absent by construction, but for the opposite reason.

Figure 7 shows that the two extreme cases move the wage offer structure in opposite directions, and that the baseline falls between them. Removing saving raises wage offers over most of the ladder, with the largest gain at the bottom (0.6% in the lowest decile). When every worker searches with the urgency of the wealth-poor, the search-adjustment discipline on low-productivity firms is maximal. The response is smaller in magnitude than that of perfect insurance. At the calibrated median buffer of 4.35 months of income, workers hold only modest self-insurance, and the baseline economy already operates closer to the no-saving case than to perfect insurance.

Table 5 summarizes the aggregate consequences. Essentially every outcome moves mono-

Table 5: Labor Market Outcomes across the Insurance Spectrum

	Baseline	Perfect insurance	Hand to mouth
Std. dev. log wage	0.20	+6.4%	-1.2%
Avg. wage	0.99	-1.1%	+0.3%
Avg. reemployment wage	0.66	-1.5%	+0.6%
Unemployment rate	0.06	+1.2 p.p.	-0.5 p.p.
JJ rate (monthly)	0.02	-0.8%	+0.0%
Avg. labor productivity	1.18	-0.6%	+0.2%

Note: This table reports aggregate labor market outcomes in the baseline economy (levels) and their changes in the perfect-insurance benchmark and in the no-saving (hand-to-mouth) economy. The unemployment rate is reported as a percentage-point change; all other changes are in percent.

tonically in the degree of insurance. Relative to the baseline, perfect insurance raises the standard deviation of log wages by 6.4%, lowers the average wage by 1.1%, and raises the unemployment rate by 1.2 p.p. as unemployed workers lose the precautionary urgency to exit unemployment. Removing saving compresses the standard deviation of log wages by 1.2%, raises the average wage by 0.3%, and lowers the unemployment rate by 0.5 p.p. The average reemployment wage changes more than the average wage, reflecting that firms at the lower end of the ladder, where workers arriving out of unemployment typically start, respond more strongly.²¹ In addition, average labor productivity, defined as output per employed worker, falls by 0.6% in the perfect-insurance economy partly because of the reduced job-to-job reallocation and partly because low-productivity firms expand by creating more vacancies, as highlighted in panel (b) of Figure 6.

Read in reverse, these numbers imply that workers' imperfectly insured exposure to job-loss risk compresses equilibrium wage inequality, and restrains wage markdowns at the bottom of the job ladder. Intuitively, uninsured risk concentrates search effort and the search-adjustment component of the labor supply elasticity on low-productivity firms. This forces low-productivity firms to offer higher wages than they otherwise would, compressing the wage distribution from below.

The results in this section have implications for social insurance and redistributive policies. In my model, self-insurance through wealth accumulation is not merely a private buffer

²¹Since the reservation wage equals b in all three economies, the reemployment (or entry) wage is simply the mean of the offer distribution $F(w)$, and its movement reflects the firm side alone rather than the worker-side acceptance margin.

against risk. It is also an equilibrium input into firms' wage-setting power. Policies that substitute for private wealth weaken the discipline that worker search behavior imposes on low-wage employers, and part of the insurance value is offset through a lower wage and a more dispersed wage distribution than a complete-markets analysis would predict.

6 The Impact of Unemployment Insurance

The previous section explored the full spectrum between zero and complete insurance against job-loss risk. This section in turn asks what this mechanism implies for the impact of a realistic policy instrument that moves actual economies along that spectrum. Specifically, I consider unemployment insurance (UI).

As far as the labor market impact is concerned, the UI literature tends to emphasize the direct disincentive channel: UI benefits affect the job search of unemployed workers by reducing the relative value of employment (e.g. [Meyer 1990](#), [Lentz 2009](#), [Birinci and See 2023](#)) and firms' job creation through an improved worker outside option (e.g. [Krusell et al. 2010](#), [Mitman and Rabinovich 2015](#)). The consumption-smoothing aspect of UI tends to be viewed in isolation as an input for the improvement in worker welfare ([Gruber 1997](#), [Chetty 2006](#), [Landaís et al. 2018](#)). However, the discussion in the previous section demonstrates that the insurance aspect itself can affect firm-side wage setting and job creation behavior, changing the aggregate labor market outcomes. The contribution of this section is to quantify what I call the insurance channel relative to the overall impact of UI.

I define the disincentive channel as the component of UI's impact that is present even under perfect insurance. I define the insurance channel as the difference between the policy responses in the incomplete-markets and perfect-insurance economies.

For this purpose, the perfect-insurance family economy in [Section 5](#) serves as a useful benchmark. I implement the increase in UI benefits in both the baseline economy and the perfect-insurance family economy, and I refer to the difference in policy responses as the insurance channel.

6.1 Policy

I interpret b in the baseline calibration as a baseline UI benefit replacing 40% of income on average, and now provide an additional benefit z_u financed by a balanced-budget uniform

Table 6: Equilibrium Impact of Unemployment Insurance

	Baseline (%)	Perfect insurance (%)
Unemployment rate	+11.3	+10.5
JJ rate	-5.9	-9.4
Average wage	+1.8	+2.8
Std. dev. log wage	-11.6	-17.5
Avg. labor productivity	+0.9	+1.5
Welfare, utilitarian (CEV)	+1.5	+1.9

Note: Responses to raising the replacement rate from 40 to 50 percent, in the baseline economy and in the perfect-insurance family economy of Section 5. I refer to the difference in policy responses as the insurance channel. The last row reports the consumption-equivalent welfare gain of the average worker in percent of lifetime consumption, net of the change in aggregate firm profits (see the footnote in the text).

tax τ on employed workers. The balanced budget requires $\tau(1 - u) = z_u u$, where u is the unemployment rate. The unemployed worker's value function becomes

$$\begin{aligned} \rho U(a) = \max_{c,s} & u(c) + U'(a)(ra + b + z_u - c) - \psi(s) \\ & + s\lambda \int \max\{W(a, w') - U(a), 0\} dF(w'), \end{aligned} \quad (49)$$

while the employed worker's problem is as in the baseline with flow income $w - \tau$. I choose z_u so that the replacement rate rises from the baseline 40% to 50% of the average wage.

Note that the benefit operates on wage posting through the outside option. Since every unemployed worker receives z_u , the common reservation wage rises to $b + z_u + \tau$, which becomes a floor under the offer distribution. Firms with productivity below the floor cannot operate. This reservation-wage channel is a standard consequence of UI in wage-posting models and is present with or without wealth.

6.2 Equilibrium Effects

Table 6 reports the results, which deliver the central message of this section: the insurance channel of UI substantially offsets the direct disincentive effects of UI on job-to-job transitions and the wage structure, while leaving the unemployment response largely unchanged. The unemployment rate rises by 11.3% of its initial level in the baseline and by 10.5% in

the perfect-insurance economy. The unemployment response reflects the relative-value and reservation-wage channels, which both economies share. The wage-structure responses, in contrast, are overstated by 50 to 60% when the insurance channel is ignored. The average wage rises by 1.8% in the baseline against 2.8% in the family economy, and the standard deviation of log wages falls by 12% against 17%. The policy also crowds out private saving, raising the wealth Gini coefficient among workers from 0.36 to 0.38.

The last row of Table 6 reports the welfare impact for the average worker as the consumption-equivalent variation (CEV), netting the change in aggregate firm profits against the gains in worker welfare.²² Consider first the perfect-insurance economy. The gain is 1.9% of lifetime consumption. Because the benefit and the offsetting tax cancel within the family budget, this gain contains no insurance value. It largely reflects the equilibrium reallocation triggered by the policy. The higher reservation wage pushes the least productive firms out of the market, employment shifts toward more productive firms, and output per worker rises. Even under complete markets, UI thus improves welfare through the firm side of the labor market.

In the baseline incomplete-markets economy, the gain is 1.5%. The welfare impact of UI is thus smaller in the incomplete-markets economy, even though this is the economy in which the policy provides actual insurance. Importantly, the firm-side response specific to the incomplete-markets economy weakens the reallocation gain. The insurance channel relaxes the search discipline of wealth-poor workers, low-wage firms cut offers relative to the perfect-insurance response, and low-productivity firms create more vacancies and retain workers that would otherwise reallocate toward more productive firms. Average labor productivity consequently rises by 0.9% against 1.5% under perfect insurance (Table 6). Wage changes themselves largely net out against profits in the welfare measure, and the smaller welfare gain therefore reflects this weaker production-efficiency gain rather than the lower wages themselves.

Although this section focuses on UI, the same logic applies to any social insurance policy that substitutes for private wealth, and even for policies without a direct search disincentive,

²²Firm profits are not rebated to workers in the model, and the policy lowers the aggregate profit flow by 0.6% of aggregate worker income in the baseline economy and by 0.9% in the perfect-insurance economy. The reported numbers subtract this change, valued as a proportional consumption flow to workers. The valuation is first order in the spirit of the envelope theorem. An actual rebate would enter household budgets, change saving and search decisions, and thereby change the equilibrium itself. Thus, there is no exactly behavior-neutral way to rebate firm profits, but the behavioral adjustments a rebate would induce have only second-order welfare effects given the small size of the transfer due to the envelope theorem.

the insurance channel could change the wage structure.

7 Conclusion

This paper develops an equilibrium job-ladder model with endogenous search intensity and wealth accumulation to study how worker-side wealth accumulation matters for firm-side wage-setting behavior. Workers accumulate assets as they move up the job ladder and draw on their wealth to smooth consumption following job loss. Since workers' search intensity responds endogenously to their wealth position, changes in the wealth distribution feed back into vacancy yields and turnover rates across firms, which in turn reshape firms' wage-setting and vacancy-creation decisions and thereby determine the equilibrium structure of the job ladder.

Using the calibrated model, I find that moving from the incomplete-markets baseline to an economy with perfect insurance induces larger wage inequality by allowing low-wage firms to offer even lower wages. The main underlying channel is that wealth-poor workers in the baseline economy are wage-sensitive, putting upward pressure on wages paid by low-productivity firms. With perfect insurance, these workers become less wage-sensitive, allowing low-productivity firms to offer lower wages.

Building on this mechanism, I quantify what incomplete markets imply for the evaluation of unemployment insurance. Raising the replacement rate from 40 to 50% raises unemployment almost identically in the baseline and in the perfect-insurance economy, but ignoring the insurance channel overstates the policy's effects on the wage structure by 50 to 60%. The insurance channel also lowers the welfare gain from the policy below its complete-markets counterpart, as the weakened wage discipline dampens the reallocation toward productive firms and the resulting efficiency loss offsets the consumption-smoothing gain.

A central takeaway is that self-insurance through wealth accumulation is not only a private buffer against risk but also an equilibrium input into firms' wage-setting behavior. Policies that substitute for private wealth weaken the discipline that worker search behavior imposes on low-wage employers, and might have unintended consequences for the equilibrium wage structure.

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Online Appendix (Not for Publication)

A Proofs

This appendix provides the proofs of the propositions in the main text and also collects several basic properties of the value functions and the policy functions that are used in the proofs.

A.1 Proof of Lemma 2.1

Proof. Step 1 (Flow balance and Little's law). Consider the stationary pool of workers employed at wage w . Its outflow density is $T(w)g_e(w)$, obtained by integrating the KF equation (13) over a ; the wealth-drift term contributes nothing because it only reallocates workers within the wage- w pool, with no flux crossing the borrowing constraint by the state-constraint boundary condition and no mass escaping as $a \rightarrow \infty$ in a stationary distribution. The inflow density $I(w)$ consists of hires from unemployment and hires from lower-paying jobs:

$$I(w) = \lambda f(w) \left[S_u + \int_b^w \bar{s}_e(\tilde{w}) g_e(\tilde{w}) d\tilde{w} \right] = V f(w) Q(w), \quad (50)$$

where the second equality uses $\lambda = qV/S$ from the constant-returns matching function and the definition of $Q(w)$. Let $\bar{D}(w)$ denote the average duration of matches entering at wage w . Little's law (Little, 1961) states that the stationary stock equals the inflow rate times the average duration of entrants, $g_e(w) = I(w)\bar{D}(w)$, and stationarity equates inflow and outflow, $I(w) = T(w)g_e(w)$. Combining the two,

$$\bar{D}(w) = \frac{g_e(w)}{I(w)} = \frac{1}{T(w)}. \quad (51)$$

Step 2 (Average entrant duration equals $D(w; 0)$). Conditional on initial wealth a , assets evolve according to $\dot{a}_t = ra_t + w - c_e(a_t, w)$ and the match ends at hazard $\tau(a_t, w) = \delta + s_e(a_t, w)\lambda(1 - F(w))$, and therefore the expected match duration is $d(a, w; 0)$. Averaging over the wealth distribution of entrants yields

$$\bar{D}(w) = \int_a^\infty d(a, w; 0) h(a | w) da = D(w; 0), \quad (52)$$

which together with (51) establishes $D(w; 0) = 1/T(w)$. $\square$

A.2 Properties of the Value Functions and the Policy Functions

Throughout this subsection, I maintain the standard smoothness conditions for HJB equations with a state-constraint boundary condition: the value functions $U(a)$ and $W(a, w)$ are continuously differentiable in a on the interior of the state space, and $W(a, w)$ is continuously differentiable in w (see Achdou et al. 2022). Statements below that involve derivatives of the value functions are understood under this maintained smoothness. Where cross derivatives such as $\partial_{aw}^2 W$ appear (in the discussion of Assumption 2 and in Proposition A.4), their existence is likewise maintained.

A.2.1 Value Functions

Lemma A.1 (Monotonicity in wealth and wage). *The value functions $U(a)$ and $W(a, w)$ are strictly increasing in wealth a for each wage w . In addition, for each a , $W(a, w)$ is strictly increasing in the current wage w .*

Proof. Monotonicity in wealth. Under the maintained smoothness, the consumption first-order conditions (6)–(7) hold throughout the state space, including at the borrowing constraint, where they hold with equality under the state-constraint boundary condition. Since $u'(c) > 0$ for every $c > 0$, they deliver $U'(a) = u'(c_u(a)) > 0$ and $\partial_a W(a, w) = u'(c_e(a, w)) > 0$. Both value functions are therefore strictly increasing in wealth.

Monotonicity in wage. Fix assets a and wages $w_1 < w_2$, and let the worker at (a, w_2) replicate the optimal plan of the worker at (a, w_1) . The income flows differ only while the initial match survives, when the replicating worker earns the additional flow $w_2 - w_1 > 0$. The asset gap thus satisfies $\dot{\Delta}_t = r\Delta_t + (w_2 - w_1)\mathbb{1}\{t < \mathcal{T}\}$ with $\Delta_0 = 0$, where $\mathcal{T}$ is the duration of the initial match, so $\Delta_t > 0$ for all $t > 0$. Feasibility follows as before, and $W(a, w_2) \geq W(a, w_1)$. For the strict inequality, let the replicating worker instead consume an additional flow $\varepsilon \in (0, w_2 - w_1)$ while the initial match survives. The gap then still grows at a positive rate during the match and remains positive afterward, and the perturbed plan is feasible. Since the initial match survives for a strictly positive duration almost surely, the perturbed plan yields strictly higher expected utility, and $W(a, w_2) > W(a, w_1)$. $\square$

Assumption 1 (Weak concavity in wealth). The value functions $U(a)$ and $W(a, w)$ are weakly concave in wealth a .

Concavity in wealth is a standard property of consumption–saving problems, but two features of the present model prevent a general proof. First, the option to accept or reject offers makes the continuation value an upper envelope of value functions, such as

$\max\{W(a, w'), U(a)\}$, and an upper envelope of concave functions need not be concave. Second, the optimized search payoff $\max_s\{-\psi(s) + s\Delta\}$ is a convex and increasing function of the employment gain Δ , while the gain itself is a difference of value functions, which need not preserve concavity. The same issue arises in [Lise \(2013\)](#), the closest environment to mine: he proves strict concavity of the value function when search effort is exogenous (his Lemma A.4), notes that with endogenous search effort concavity has not been established in general, and confirms it in his numerical solutions instead. I follow the same approach and state concavity as an assumption. Given the consumption first-order conditions, weak concavity of the value functions is equivalent to consumption being weakly increasing in wealth, and the numerical verification amounts to checking the monotonicity of the consumption policies, which holds in all numerical solutions in this paper, as confirmed in [Appendix A.2.4](#).

A.2.2 Consumption Policy Functions

Lemma A.2 (Consumption monotonicity in wealth). *Under Assumption 1, the optimal consumption policies $c_u(a)$ and $c_e(a, w)$ are weakly increasing in assets a for each w .*

Proof. The consumption first-order conditions (6) and (7) give

$$u'(c_u(a)) = U'(a), \quad u'(c_e(a, w)) = \partial_a W(a, w).$$

By Assumption 1, the marginal values of assets, $U'(a)$ and $\partial_a W(a, w)$, are weakly decreasing in a . Since u' is strictly decreasing, $c_u(a)$ and $c_e(a, w)$ are weakly increasing in a . $\square$

Finally, I make a second assumption about the dependence of consumption on the wage.

Assumption 2 (Consumption monotonicity in wage). For each a , the optimal consumption policy $c_e(a, w)$ is weakly increasing in wage w .

Assumption 2 is natural but technically demanding to prove in the continuous-time setting. Differentiating the first-order condition $u'(c_e(a, w)) = \partial_a W(a, w)$ with respect to w shows that the assumption is equivalent to $W(a, w)$ having decreasing differences in (a, w) , that is, $\partial_{aw}^2 W(a, w) \leq 0$. Establishing such a second-order property of the solution to an HJB equation with Poisson jumps and a borrowing constraint is beyond available comparative-statics results, and even in closely related heterogeneous-agent models consumption monotonicity in income is typically imposed or verified numerically rather than proved analytically. For this reason, I state Assumption 2 explicitly and confirm that it holds in numerical solutions.

A.2.3 Search Policy Functions

Define the expected gains from search as

$$\Delta_u(a) = \lambda \int_b^{\bar{w}} (W(a, w) - U(a)) dF(w), \quad (53)$$

$$\Delta_e(a, w) = \lambda \int_w^{\bar{w}} (W(a, w') - W(a, w)) dF(w'), \quad (54)$$

so that the search first-order conditions (10)–(11) read $\psi'(s_u(a)) = \Delta_u(a)$ and $\psi'(s_e(a, w)) = \Delta_e(a, w)$. The max operators in the first-order conditions are dropped and the integrals start at b and w because $W(a, \cdot)$ is increasing (Lemma A.1) and $U(a) = W(a, b)$ by the reservation-wage property (5). Since ψ is strictly convex, ψ' is strictly increasing and invertible on its range. The policies satisfy

$$s_u(a) = (\psi')^{-1}(\Delta_u(a)), \quad s_e(a, w) = (\psi')^{-1}(\Delta_e(a, w)). \quad (55)$$

Proposition A.3 (Monotonicity of search intensity). *Under Assumption 2, the optimal search policies $s_u(a)$ and $s_e(a, w)$ are weakly decreasing in wealth a . The employed search policy $s_e(a, w)$ is weakly decreasing in the current wage w without further assumptions.*

Proof. Equation (55) implies that the search policies are increasing transformations of the gains $\Delta_u(a)$ and $\Delta_e(a, w)$ since $(\psi')^{-1}$ is increasing. It is therefore enough to sign the derivatives of the gains. Differentiating $\Delta_u(a)$ and $\Delta_e(a, w)$ with respect to a , combined with the first-order conditions for consumption (6) and (7), gives

$$\Delta'_u(a) = \lambda \int_b^{\bar{w}} (u'(c_e(a, w)) - u'(c_u(a))) dF(w) \leq 0, \quad (56)$$

$$\partial_a \Delta_e(a, w) = \lambda \int_w^{\bar{w}} (u'(c_e(a, w')) - u'(c_e(a, w))) dF(w') \leq 0. \quad (57)$$

Note that I have $c_u(a) = c_e(a, b)$ from differentiating $U(a) = W(a, b)$ (the reservation-wage property (5)) with respect to a and applying the consumption first-order conditions. Each integrand therefore compares consumption at two ranked wages and is nonpositive by Assumption 2. These inequalities imply that $s_u(a)$ and $s_e(a, w)$ are weakly decreasing in a . Differentiating $\Delta_e(a, w)$ with respect to w gives

$$\partial_w \Delta_e(a, w) = -\lambda(1 - F(w))\partial_w W(a, w) \leq 0, \quad (58)$$

where $\partial_w W(a, w) \geq 0$ because $W(a, w)$ is increasing in w by Lemma A.1. This inequality implies that $s_e(a, w)$ is weakly decreasing in w . $\square$

Finally, I explore how the wage sensitivity of on-the-job search depends on wealth. For this result, I impose the iso-elastic form $\psi(s) = \psi_0 s^{1+\eta_s}/(1+\eta_s)$ and define the individual search semi-elasticity

$$\epsilon_{s,w}(a, w) = -\partial_w \log s_e(a, w). \quad (59)$$

This is the worker-level counterpart of the semi-elasticity of the average search intensity entering the markdown decomposition in Section 3.2.

Proposition A.4 (Wealth and the wage sensitivity of search). *Suppose $\psi(s) = \psi_0 s^{1+\eta_s}/(1+\eta_s)$. Let $m(a, w) = \partial_w W(a, w)$ denote the marginal value of the wage. Then the following hold for $w < \bar{w}$, where the employment gain $\Delta_e(a, w)$ is strictly positive:*

(i) *The search semi-elasticity satisfies*

$$\epsilon_{s,w}(a, w) = \frac{1}{\eta_s} \frac{(1 - F(w)) m(a, w)}{\int_w^{\bar{w}} (1 - F(x)) m(a, x) dx}. \quad (60)$$

(ii) *Its wealth dependence is given by*

$$\partial_a \log \epsilon_{s,w}(a, w) = \partial_a \log m(a, w) - \int_w^{\bar{w}} \partial_a \log m(a, x) d\mu_{a,w}(x), \quad (61)$$

where $\mu_{a,w}$ is the distribution over $(w, \bar{w})$ with density proportional to $(1 - F(x)) m(a, x)$.

(iii) *If m is log-supermodular in (a, w) , then $\epsilon_{s,w}(a, w)$ is weakly decreasing in wealth a .*

Proof. Part (i). The search first-order condition yields $s_e(a, w) = (\Delta_e(a, w)/\psi_0)^{1/\eta_s}$. I therefore have

$$\epsilon_{s,w}(a, w) = -(1/\eta_s) \frac{\partial_w \Delta_e(a, w)}{\Delta_e(a, w)}. \quad (62)$$

Note that the numerator is $\partial_w \Delta_e = -\lambda(1 - F(w))m(a, w)$ as in the proof of Proposition A.3. For the denominator, integration by parts yields

$$\Delta_e(a, w) = \lambda \int_w^{\bar{w}} (W(a, x) - W(a, w)) dF(x) = \lambda \int_w^{\bar{w}} (1 - F(x)) m(a, x) dx, \quad (63)$$

where the boundary terms vanish because $W(a, x) - W(a, w) = 0$ at $x = w$ and $1 - F(\bar{w}) = 0$. Substituting the two expressions yields (60).

Part (ii). Taking logs of (60) and differentiating with respect to a gives

$$\partial_a \log \epsilon_{s,w}(a, w) = \partial_a \log m(a, w) - \partial_a \log \left(\int_w^{\bar{w}} (1 - F(x)) m(a, x) dx \right). \quad (64)$$

The last term equals

$$\frac{\int_w^{\bar{w}} (1 - F(x)) m(a, x) \partial_a \log m(a, x) dx}{\int_w^{\bar{w}} (1 - F(x)) m(a, x) dx},$$

which is the average of $\partial_a \log m(a, \cdot)$ with density proportional to $(1 - F(x))m(a, x)$.

Part (iii). The measure $\mu_{a,w}$ is supported on $x > w$, and therefore when $\partial_a \log m(a, \cdot)$ is nondecreasing, the average in (61) weakly exceeds $\partial_a \log m(a, w)$ and the right-hand side of (61) is nonpositive. $\square$

Two remarks help interpret the proposition. First, applying the envelope theorem to the HJB equation (4) gives $(\rho + \tau(a, w)) m(a, w) = u'(c_e(a, w)) + (ra + w - c_e(a, w)) \partial_a m(a, w)$, where $\tau(a, w) = \delta + s_e(a, w)\lambda(1 - F(w))$ is the separation hazard, and the solution of this equation along the asset path is

$$m(a, w) = \int_0^\infty \exp\left(-\rho t - \int_0^t \tau(a_k, w) dk\right) u'(c_e(a_t, w)) dt. \quad (65)$$

The marginal value of the wage is the expected discounted marginal utility of consumption over the remaining duration of the match, the marginal-utility-weighted counterpart of the expected discounted duration $d(a, w; \rho_f)$ in equation (22).

Second, the log-supermodularity condition in part (iii) is a shape restriction that goes beyond Assumption 2. By the consumption first-order conditions, Assumption 2 is equivalent to $\partial_a m \leq 0$, i.e., the *level* of the marginal value of the wage falls with wealth. Part (iii) additionally restricts the profile of this decline. Since wealth lowers $m(a, w)$ everywhere, log-supermodularity requires that wealth erode the marginal value of the wage proportionally fastest at the bottom of the ladder. The condition is thus the formal statement that wealth and the wage are complements in the log marginal value of wage income, with private wealth substituting most strongly for wage income at the bottom rung. This asymmetry, rather than the overall decline in marginal utility, is what makes the wage sensitivity of search fall with wealth, since a proportional shift of the entire profile $m(a, \cdot)$ cancels between the numerator and the denominator of (60). This shape restriction is a second-order property of the value function that makes analytical proof challenging, and I verify its implication numerically in Appendix A.2.4: in the estimated model, $\epsilon_{s,w}(a, w)$ is decreasing in wealth

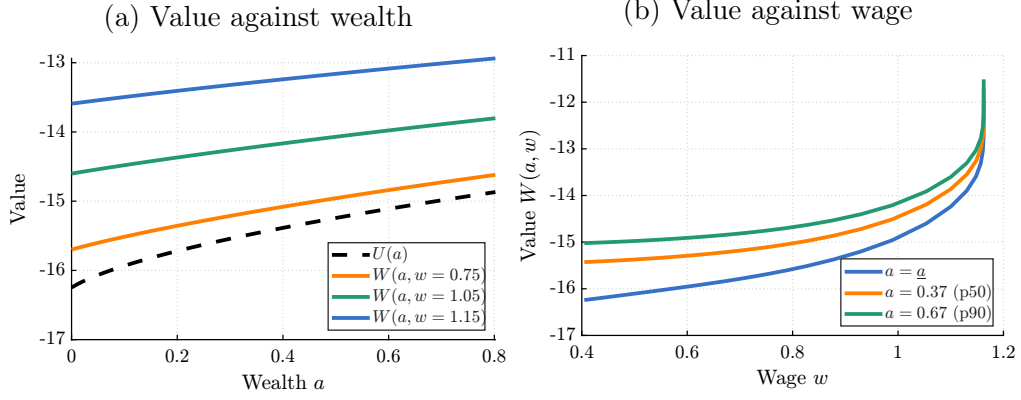
at essentially every point of the state space. The gradient is quantitatively large where it matters: at the bottom rungs of the ladder, the semi-elasticity of a worker at the borrowing constraint is roughly twice that of a worker at the 90th percentile of the wealth distribution, and the gap fades toward the top of the ladder, consistent with equation (61), whose right-hand side vanishes as $\mu_{a,w}$ concentrates near w . The calibrated search cost includes the costless floor $\underline{s}$; the proposition then applies to the costly part of search $s_e - \underline{s}$, and the numerical monotonicity holds both for $s_e - \underline{s}$ and for s_e itself.

A.2.4 Numerical Confirmation

Figures 8–12 confirm the statements of this subsection in the model solved at the estimated parameters; each figure displays representative wage rungs or wealth levels, and every monotonicity shown also holds grid point by grid point over the entire state space, with the single exception of the semi-elasticity at the very top of the ladder, noted below. There is one remark: the quantitative model incorporates the extensions described in Section 4 (unsolicited offers through the costless search floor $\underline{s}$ and the productivity-dependent destruction rate $\delta(y)$), and the solved environment therefore deviates slightly from the one underlying the formal statements above. The figures should therefore be read as confirming that the properties continue to hold under these extensions.

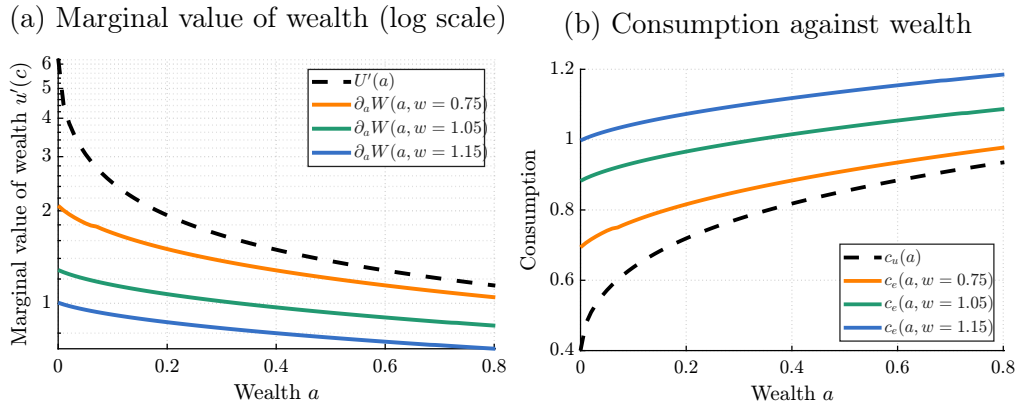
Figure 8 shows that the value functions are strictly increasing in wealth and in the wage (Lemma A.1), where the value increases steeply toward the highest wage because the job destruction rate $\delta(y)$ falls toward high-productivity firms. Panel (a) of Figure 9 shows that the marginal value of wealth, computed as $u'(c)$ via the consumption first-order conditions, is decreasing in wealth, confirming Assumption 1; equivalently, panel (b) shows that consumption is increasing in wealth (Lemma A.2). Figure 10 shows that consumption is increasing in the current wage (Assumption 2). Figure 11 shows that search intensity is decreasing in wealth for both unemployed and employed workers and decreasing in the current wage (Proposition A.3). Figure 12 shows that the search semi-elasticity $\epsilon_{s,w}(a, w)$ is decreasing in wealth, with the steepest gradient at the bottom of the ladder, and that the wealth gap fades toward the top, confirming part (iii) of Proposition A.4 and the concentration logic of equation (61); the only grid points where this monotonicity fails numerically lie at the very top of the ladder, where the employment gain vanishes and the semi-elasticity diverges.

Figure 8: Value Functions: Monotonicity in Wealth and Wage



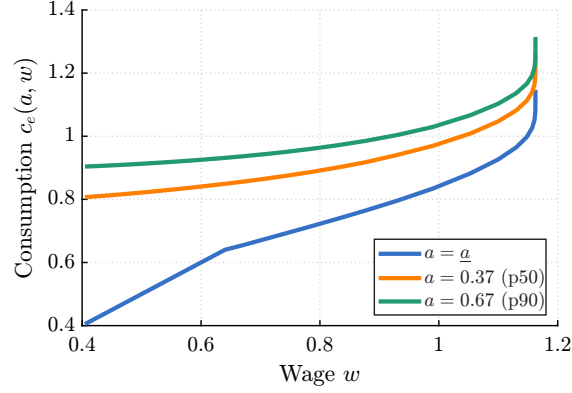
Note: Panel (a) displays $U(a)$ and $W(a, w)$ at selected wage rungs against wealth. Panel (b) displays $W(a, w)$ against the wage at the borrowing constraint and at the median and 90th percentile of the stationary wealth distribution. Wealth axes are truncated at the 99.9th percentile of the stationary wealth distribution.

Figure 9: Concavity in Wealth and the Consumption Policy



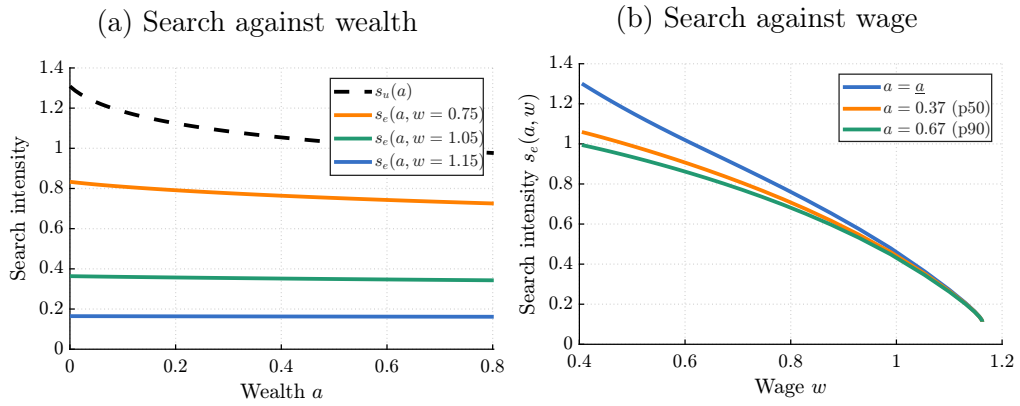
Note: Panel (a) displays the marginal value of wealth $U'(a)$ and $\partial_a W(a, w)$, computed as $u'(c)$ via the consumption first-order conditions, on a log scale. Panel (b) displays the corresponding consumption policies. By the first-order conditions, the two panels are equivalent statements of Assumption 1 and Lemma A.2.

Figure 10: Consumption Policy: Monotonicity in the Wage



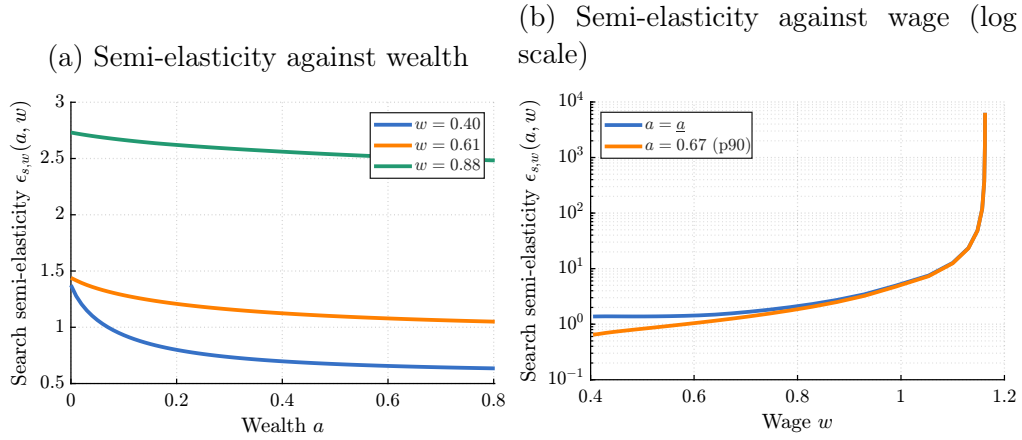
Note: Consumption of employed workers against the wage, at the borrowing constraint and at the median and 90th percentile of the stationary wealth distribution.

Figure 11: Search Policy: Monotonicity in Wealth and Wage



Note: Panel (a) displays the search policies $s_u(a)$ and $s_e(a, w)$ at selected wage rungs against wealth. Panel (b) displays $s_e(a, w)$ against the wage at selected wealth levels.

Figure 12: Search Semi-Elasticity: Dependence on Wealth



Note: The semi-elasticity $\epsilon_{s,w}(a, w)$ is computed as $-\partial_w \log(s_e - \underline{s})$ on the wage rungs where $s_e - \underline{s}$ is bounded away from zero; the top of the ladder is dropped because the employment gain vanishes there. Panel (a) displays the semi-elasticity against wealth at bottom, low, and middle rungs. Panel (b) displays it against the wage, on a log scale, at the borrowing constraint and at the 90th percentile of the stationary wealth distribution.

A.3 Derivation of the Wage Function

The first-order condition for the wage function is

$$w'(y) = (y - w(y)) \cdot \frac{n'(y)}{n(y)}. \quad (66)$$

Rewriting this equation yields

$$n(y)w'(y) + n'(y)w(y) = yn'(y). \quad (67)$$

Rewriting the left-hand side gives

$$[n(y)w(y)]' = yn'(y). \quad (68)$$

Integrating both sides from b to y and using the boundary condition $w(b) = b$ yields

$$n(y)w(y) - n(b)b = \int_b^y xn'(x)dx, \quad (69)$$

which gives

$$w(y) = b \frac{n(b)}{n(y)} + \int_b^y x \frac{n'(x)}{n(y)} dx. \quad (70)$$

Another useful expression is as follows. Integration by parts gives

$$\int_b^y xn'(x)dx = yn(y) - bn(b) - \int_b^y n(x)dx. \quad (71)$$

Substituting this expression into the previous equation yields

$$w(y) = y - \int_b^y \frac{n(x)}{n(y)} dx. \quad (72)$$

This corresponds to equation (37).

A.4 Derivation of the Member Values in the Family Economy

This appendix derives the member values (46)–(47) from the family's planning problem in Section 5.1. The equations are envelope conditions of that problem, and $V(w)$ and V_u are shadow values of members converted into units of income.

The family's state consists of its assets A and the distribution of members over labor-

market states, namely the unemployed mass u and the density $g_e(w)$ of members employed at wage w . Given the family's search policies s_u and $s_e(\cdot)$, the member distribution evolves according to

$$\dot{g}_e(w) = u s_u \lambda f(w) + \lambda f(w) \int_b^w s_e(x) g_e(x) dx - [\delta + s_e(w) \lambda (1 - F(w))] g_e(w), \quad (73)$$

$$\dot{u} = \delta \int_b^{\bar{w}} g_e(w) dw - u s_u \lambda (1 - F(b)). \quad (74)$$

Let $\mathcal{V}(A, u, g_e)$ denote the family's value. Writing $\Omega_u = \partial \mathcal{V} / \partial u$ and $\Omega_e(w) = \partial \mathcal{V} / \partial g_e(w)$ for the marginal values of members, where the latter is a functional derivative, the family's HJB equation reads

$$\begin{aligned} \rho \mathcal{V}(A, u, g_e) = & \max_{C, s_u, s_e(\cdot)} u(C) - u \psi(s_u) - \int_b^{\bar{w}} \psi(s_e(w)) g_e(w) dw \\ & + \partial_A \mathcal{V}(A, u, g_e) \dot{A} + \Omega_u \dot{u} + \int_b^{\bar{w}} \Omega_e(w) \dot{g}_e(w) dw, \end{aligned} \quad (75)$$

with $\dot{A} = rA + b u + \int w g_e(w) dw - C$ and the flows (73)–(74). The first-order condition for consumption is $u'(C) = \partial_A \mathcal{V}(A, u, g_e)$.

Differentiating (75) with respect to $g_e(w)$ and applying the envelope theorem yields, in steady state where the shadow values are constant,

$$\rho \Omega_e(w) = \max_s -\psi(s) + u'(C) w + s \lambda \int_w^{\bar{w}} (\Omega_e(w') - \Omega_e(w)) dF(w') + \delta (\Omega_u - \Omega_e(w)). \quad (76)$$

The four terms collect every appearance of $g_e(w)$ in (75), namely the member's search cost, her income contribution valued at the marginal value of wealth, her moves up the ladder valued at shadow-value differences, and her transition to unemployment. The destruction inflow in (74) contributes the term $\delta \Omega_u$, and the poaching inflow in (73) evaluated at higher wages contributes $s \lambda \int_w^{\bar{w}} \Omega_e(w') dF(w')$. Differentiating with respect to u instead gives the analogous condition with flow income b and offers accepted from b upward.

Dividing (76) by $u'(C)$, which is constant in steady state, and defining $V(w) = \Omega_e(w) / u'(C)$ and $V_u = \Omega_u / u'(C)$ delivers (46)–(47). The first-order condition for search, $\psi'(s) = \lambda \int_w^{\bar{w}} (\Omega_e(w') - \Omega_e(w)) dF(w')$, is the family's search rule reported in the text. Since the member equations are linear in the values given policies, integrating forward gives the representation $V(w) = \mathbb{E} \int_0^\infty e^{-\rho t} (inc_t - \psi(s_t) / u'(C)) dt$ along the member's career path, which is the interpretation of V as the expected present value of the income the member contributes to the

pool, net of the income equivalent of her search costs.

Two remarks connect the derivation to the computations. First, with the exit extension of Section 4, the flows (73)–(74) acquire the outflows $-\mu g_e(w)$ and $-\mu u$, and the envelope conditions carry the effective discount $\rho + \mu$, while the newborn inflow is a constant mass independent of the distribution and therefore does not appear in the member values. Second, the derivation uses the steady state twice, once for the constancy of the shadow values and once for the constancy of $u'(C)$. Along a transition, the member equations would gain a time-derivative term and a time-varying conversion rate.

B The Full Quantitative Model

The quantitative analysis in Sections 4–6 augments the model of Section 2 with the two extensions introduced in Section 4: perpetual-youth worker entry and exit, and a job destruction rate that depends on firm productivity. This appendix states the full model and records exactly where each equilibrium condition changes. Setting $\mu = 0$ and a flat profile $\delta(y) = \delta$ recovers the model of Section 2 equation by equation.

Demographics. Workers exit the labor market at rate μ , independently of their employment state, and the value of exit is normalized to zero. Each exiting worker is replaced by a newborn who enters unemployment with zero wealth, and the population is stationary with measure one. There are no annuity markets, and the assets of exiting workers are not redistributed to surviving workers. From the firm’s perspective, a worker’s exit is a separation, and μ enters firm-level turnover below.

Destruction profile. The job destruction rate is a decreasing function $\delta(y)$ of the employer’s productivity. Because the equilibrium wage schedule $w(y)$ is strictly increasing (rank preservation extends to this environment, as shown below), a worker’s wage reveals the employer’s productivity in equilibrium, and I write

$$\tilde{\delta}(w) = \delta(y(w)), \tag{77}$$

where $y(\cdot)$ is the inverse of the equilibrium wage schedule. The worker’s state remains (a, w) .

Worker problem. Exit at rate μ to a zero-value state is equivalent to additional discounting, so the HJB equations (3)–(4) become

$$(\rho + \mu) U(a) = \max_{c,s} u(c) - \psi(s) + U'(a) (ra + b - c) + s\lambda \int_{\underline{w}}^{\bar{w}} \max\{W(a, w) - U(a), 0\} dF(w), \quad (78)$$

$$(\rho + \mu) W(a, w) = \max_{c,s} u(c) - \psi(s) + \partial_a W(a, w) (ra + w - c) + s\lambda \int_{\underline{w}}^{\bar{w}} \max\{W(a, w') - W(a, w), 0\} dF(w') + \tilde{\delta}(w) (U(a) - W(a, w)). \quad (79)$$

The consumption and search first-order conditions (6)–(11) and the state-constraint boundary conditions are unchanged in form. The reservation wage is also the same. At $w = \underline{w}(a)$, the destruction term $\tilde{\delta}(w)(U(a) - W(a, w))$ vanishes regardless of the level of $\tilde{\delta}$, and μ tilts the discount rate symmetrically in both states. Therefore, the argument for $\underline{w}(a) = b$ works as before. The destruction profile matters for behavior only through the value function. Low rungs of the ladder carry both low wages and high destruction risk, which steepens the value gains from climbing.

Worker distributions. The KF equations (12)–(13) acquire the exit outflow μ , the wage-specific destruction rate, and the newborn inflow. For $a > \underline{a}$,

$$0 = -\partial_a \left((ra + b - c_u(a)) g_u(a) \right) - s_u(a) \lambda (1 - F(b)) g_u(a) + \int_b^{\bar{w}} \tilde{\delta}(w) g_e(a, w) dw - \mu g_u(a), \quad (80)$$

$$0 = -\partial_a \left((ra + w - c_e(a, w)) g_e(a, w) \right) - \left[\tilde{\delta}(w) + \mu + s_e(a, w) \lambda (1 - F(w)) \right] g_e(a, w) + s_u(a) \lambda f(w) \mathbb{1}\{w \geq b\} g_u(a) + \lambda f(w) \int_b^w s_e(a, x) g_e(a, x) dx. \quad (81)$$

Newborns enter as a point inflow of mass μ into unemployment at $a = 0$: the distribution G_u carries the corresponding mass at zero wealth, which the discretized generator implements as an inflow into the boundary node. Total population remains one, so the adding-up condition (14) is unchanged.

Firm problem. A firm with productivity y posting wage $\tilde{w}$ faces turnover

$$T(\tilde{w}; y) = \delta(y) + \mu + \bar{s}_e(\tilde{w}) \lambda (1 - F(\tilde{w})), \quad (82)$$

and its stationary size per vacancy is $\tilde{n}(\tilde{w}; y) = Q(\tilde{w})/T(\tilde{w}; y)$, where the vacancy yield Q is defined in (23) and depends only on the posted wage. The static profit problem and the vacancy first-order condition (35) are unchanged with $\tilde{n}(w(y); y)$ in place of $\tilde{n}(w)$. The wage first-order condition changes slightly. The derivative holds the firm's own $\delta(y)$ fixed, whereas the equilibrium schedule $\tilde{n}(y) = \tilde{n}(w(y); y)$ varies with y partly through the exogenous destruction profile. Differentiating along the schedule,

$$\frac{d \log \tilde{n}(y)}{dy} = \partial_{\tilde{w}} \log \tilde{n}(w(y); y) w'(y) - \frac{\delta'(y)}{T(y)},$$

where $T(y) = T(w(y); y)$, and substituting this into the first-order condition $(y - w) \partial_{\tilde{w}} \log \tilde{n} = 1$ yields the corrected wage ODE

$$w'(y) = (y - w(y)) \left[\frac{d \log \tilde{n}(y)}{dy} + \frac{\delta'(y)}{T(y)} \right]. \quad (83)$$

Because Q depends only on the posted wage, the δ'/T term is the entire correction. With the calibrated declining profile ($\delta' < 0$), the correction is negative: it strips out the part of the size–productivity gradient that reflects exogenously lower destruction at better firms, and thus flattens the wage schedule relative to the uncorrected ODE.

Rank preservation. The argument of Section 2 extends. The log objective is $\log(y - \tilde{w}) + \log Q(\tilde{w}) - \log T(\tilde{w}; y)$, and since T is additively separable in $\delta(y)$ and the wage-dependent quit margin, its cross-partial in $(y, \tilde{w})$ is

$$\frac{\partial^2 \log \pi}{\partial y \partial \tilde{w}} = \frac{1}{(y - \tilde{w})^2} + \frac{\partial_{\tilde{w}} T \cdot \delta'(y)}{T^2} > 0,$$

because $\partial_{\tilde{w}} T < 0$ (higher wages improve retention) and $\delta'(y) \leq 0$. The declining destruction profile therefore reinforces, rather than threatens, the strict log-supermodularity that delivers rank preservation.

Equilibrium. The definition of a stationary equilibrium in Section 2.4 carries over with the following replacements: worker optimality refers to (78)–(79); worker distributions to

(80)–(81) with the newborn entry mass; and firm optimality to the vacancy condition (35) together with the corrected wage condition (83), with turnover (82). The offer distribution (30), the aggregation conditions, and the matching block are unchanged.

C Computation Algorithm

This appendix describes the algorithm used to compute the stationary equilibrium. The equilibrium has the following structure. The general-equilibrium objects are the wage and vacancy schedules $\{w(y), v(y)\}$, the implied wage-offer distribution F , and the contact rates (λ, q) . Given these objects, the worker block delivers the policy functions and the stationary distributions, which aggregate to the schedules S_u , $\bar{s}_e(w)$, $Q(w)$, and $T(w)$ that the firm block takes as given. The firm block in turn updates $\{w(y), v(y)\}$ and hence F , and the matching block updates (λ, q) . The equilibrium is the fixed point of this loop, summarized in Algorithm 1; each block is described below.

Discretization. Wealth is discretized on a uniform grid of 400 nodes on $[0, 4]$, and firm productivity on a uniform grid of 2,000 nodes on $[b, \bar{y}]$. The wage-offer distribution is approximated by $n_w = 30$ blocks: the support of the current F is partitioned into blocks, and employed workers are tracked by the block of their wage. This reduces the worker state space to the wealth grid times $n_w + 1$ labor-market states (unemployment plus n_w rungs of the ladder), which keeps the worker block small even though the firm side is solved on the fine productivity grid.

Worker block. Given (F, λ) , the HJB equations (3)–(4) are solved by an implicit upwind finite-difference scheme following Achdou et al. (2022). The consumption and search policies are obtained pointwise from the first-order conditions (6)–(11), and the state-constraint boundary conditions (8)–(9) are imposed at the borrowing constraint. Each implicit update solves a linear system with time step Δ . Convergence yields the discretized generator of the joint wealth–employment process under the optimal policies. The stationary distribution then solves the discretized KF equations (12)–(13) using the adjoint of the converged generator, with the demographic flow of Section 4 (exit at rate μ , newborns entering unemployment at the borrowing constraint). Aggregating the policies over the stationary distribution delivers S_u , $\bar{s}_e(w)$, the vacancy yield $Q(w)$, and the turnover rate $T(w)$.

Algorithm 1 Stationary equilibrium

Guess the firm-side schedules $Q(w), T(w)$, the offer distribution F , and the contact rates (λ, q)

repeat

Firm block: solve the wage ODE for $w(y)$ and the vacancy FOC for $v(y)$; update F via (30)

Worker block: given (F, λ) , solve the HJB equations by the implicit upwind scheme and the KF equations by the adjoint of the generator; aggregate to $S_u, \bar{s}_e(w), Q(w), T(w)$

Matching block: update $S, V, \theta = V/S$, and (λ, q) from the matching function

until the wage schedule converges in relative sup norm (tolerance 10^{-6})

Firm block. Given $Q(w)$ and $T(w)$, the stationary firm size per vacancy is $\tilde{n}(w) = Q(w)/T(w)$. The wage schedule solves the first-order condition (36) as a first-order ODE in productivity, $w'(y) = (y - w(y)) \tilde{n}'(y)/\tilde{n}(y)$ with $\tilde{n}(y) = \tilde{n}(w(y))$, integrated upward from the boundary condition $w(b) = b$ on the productivity grid; under the calibrated $\delta(y)$ profile, the ODE includes the correction term described in Section 4. The vacancy policy follows pointwise from the first-order condition (35), which is in closed form given the iso-elastic vacancy cost. Aggregating $v(y)$ across firms yields aggregate vacancies V and the updated offer distribution F via (30). Because F , and hence T and $\tilde{n}$, depend on the vacancy policy, the firm block iterates internally on (w, v, F) to convergence, with its own relaxation weight of 0.1 and tolerance 10^{-9} .

Matching block, dampening, and convergence. Aggregate search intensity S and vacancies V update the tightness $\theta = V/S$ and the contact rates $\lambda = \mathcal{M}(1, \theta)$ and $q = \mathcal{M}(\theta^{-1}, 1)$. To stabilize the fixed point, every updated object is dampened with a relaxation weight of one half. The loop terminates when the wage schedule converges in the relative sup norm with tolerance 10^{-6} . In practice, the equilibrium is found in a few dozen iterations, and the total computation time is about ten seconds.

D Data

This appendix describes the two household surveys used in the empirical analysis: the Survey of Income and Program Participation (SIPP), from which the labor market transition rates and the liquid-wealth measures are constructed, and the National Longitudinal Survey of Youth 1997 (NLSY97), which provides the direct measure of on-the-job search used in Appendix E.

D.1 SIPP Sample

Sources. I use the 1996, 2001, 2004, and 2008 SIPP panels, drawing the monthly core files and the topical module on wealth (Module 3 or 4, depending on the panel). The core files provide monthly labor-force information, employer identifiers, earnings, and demographic characteristics. The topical module provides detailed information on assets and liabilities at a single point during each panel. For each panel, I merge the monthly core files with the topical module using the panel, household, and person identifiers, so that the wealth observation is assigned to all monthly observations of the same individual.

Sample. The sample is restricted to individuals aged 25–55, male workers, private-sector wage and salary workers, and workers with strictly positive monthly earnings. I further restrict the sample to the wave in which wealth information is collected. All results are weighted using the SIPP person weights.

Labor market states and transitions. Monthly employment status is defined from the monthly employment status recode: employed, unemployed, and out of the labor force. An EU transition is a move from employment to unemployment between two consecutive months, and a UE transition is the reverse. A job-to-job transition is defined as a move in which an individual is employed in two consecutive months and the employer identifier changes, excluding cases in which the employer identifier is missing in either month. Monthly labor earnings are total person-level earnings, and observations with zero or missing earnings are dropped.

Wealth. Following [Chetty \(2008\)](#), liquid wealth is defined as total household net worth minus home equity, business equity, and vehicle equity. I additionally exclude retirement accounts, so that the measure captures the liquid buffer available to smooth consumption before retirement. Net liquid wealth subtracts unsecured debt. All monetary variables are converted into 2015 dollars using the CPI-U.

Quantiles and seam correction. Wealth and earnings quantiles are computed within each calendar year–month cell using person weights, so that quantile assignments are not confounded by panel composition or price-level differences across months. Because SIPP interviews cover four reference months at a time, transitions are disproportionately recorded at the interview seam; transition regressions therefore include interview-seam fixed effects, defined as the position of the month within the interview wave.

D.2 NLSY97 Sample

Sources. The NLSY97 follows a cohort born in 1980–1984. I use the ten survey rounds from 2008 to 2023 (annual through 2011, biennial thereafter) in which respondents are asked whether they have done anything to look for work in the past three months. The question is asked only of respondents who hold a job at the interview date and whose current job has lasted at least three months. Over these rounds the cohort ages from 24 to 43, and therefore the NLSY97 sample covers the younger half of the age range of the SIPP sample.

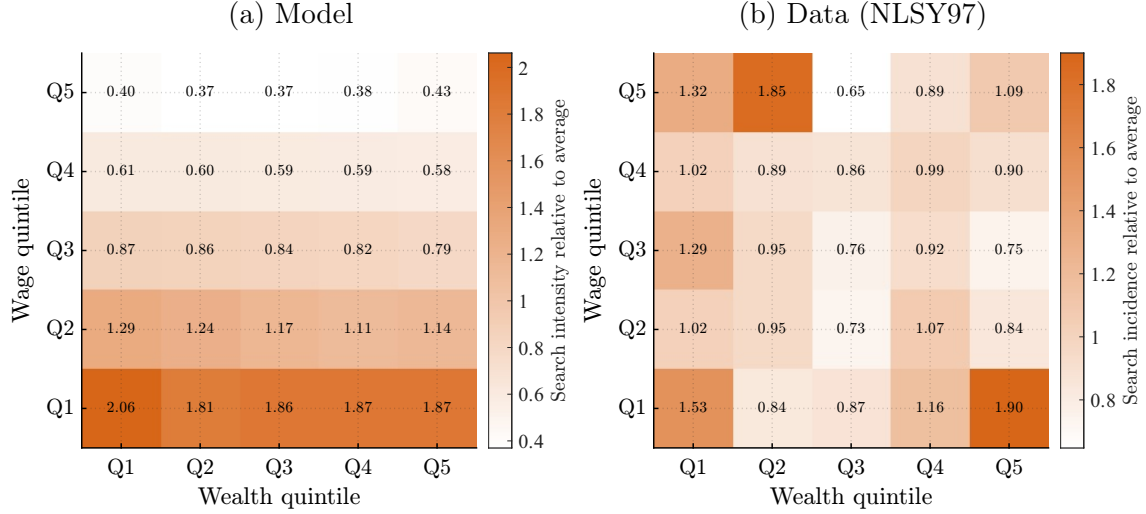
Sample. Mirroring the SIPP selection, I keep male respondents aged 25–55 who are not enrolled in school, are currently employed in the sense above, and report an hourly wage at the current job between \$3 and \$200 in 2015 dollars. I further require a valid liquid-wealth observation and a positive panel weight.

Variable construction. The search indicator equals one if the respondent reports any job search activity in the past three months. The wage is the constructed hourly rate of pay at the current (most recent) job. Liquid wealth is financial assets minus unsecured debt from the NLSY97 wealth modules, which are recorded when a respondent turns 25, 30, 35, and 40; each person-year observation is assigned the wealth observation from the nearest module age. This measure parallels the SIPP concept: it excludes housing, business, and vehicle equity, and nets out unsecured debt. As in the SIPP, all monetary variables are converted into 2015 dollars using the CPI-U: wages are deflated by the survey year, and module wealth by the year the module was administered, which differs across respondents within a survey round because module timing follows the respondent’s own age. Wealth and wage quintiles are computed within each survey round using panel weights. All statistics are weighted by the round-specific panel weights.

E On-the-Job Search and Wealth in the NLSY97

The worker-side block of the model predicts that, conditional on the current wage, wealth-poor workers search harder than wealth-rich workers (Proposition A.3). This appendix checks that prediction directly using the NLSY97 on-the-job search question described in Appendix D.2. The NLSY97 measures search on the extensive margin (whether any search activity occurred in the past three months), whereas the model’s s_e is a continuous intensity. Therefore, the comparison is about the direction and relative size of the wealth gradient

Figure 13: On-the-Job Search by Wealth and Wage: Model vs. NLSY97



Note: Each panel reports search in each wealth-quintile $\times$ wage-quintile cell, normalized by its own aggregate average, so a cell value of 1 means search at the economy-wide (sample-wide) mean. Panel (a) is the mass-weighted average search intensity s_e of employed workers in the calibrated model. Panel (b) is the weighted incidence of on-the-job search in the NLSY97 sample of Appendix D.2, relative to the sample mean of 10.2 percent; quintiles are computed within survey round using panel weights.

rather than levels.

Figure 13 shows the wealth–wage heatmap of search in the model and in the data, normalizing each panel by its own average since the extensive-margin incidence in the data and the search intensity in the model are measured in different units. The comparison makes two points. First, in both panels search is elevated in the bottom wealth quintile conditional on the wage: the bottom-wealth cell exceeds its row average in every wage quintile in the data, as it does in the model. Second, the model’s dominant variation runs along the wage dimension, whereas the extensive-margin incidence in the data varies much less across wage quintiles. This is expected since a binary indicator of any search activity in three months compresses differences in search intensity. The wage-gradient prediction is instead disciplined directly by the search-elasticity target of Faberman et al. (2022) in the estimation (Appendix F).

The heatmap should nevertheless be read with caution. Several cells, especially in the top wage rows, contain a small number of observations and are correspondingly noisy, and the raw cells do not adjust for composition. Wealth in the NLSY97 is measured at the age modules and rises mechanically with age while search declines with it. The regressions below address both concerns, pooling the wealth gradient across wage cells and adding demographic controls.

Table 7: On-the-Job Search and Liquid Wealth in the NLSY97

	(1)	(2)	(3)
Wealth Q1 (poorest)	0.022* (0.013)	0.025* (0.013)	0.029** (0.013)
Wealth Q2	-0.004 (0.011)	-0.001 (0.012)	0.028** (0.013)
Wealth Q3	-0.023** (0.011)	-0.020* (0.012)	0.009 (0.012)
Wealth Q4	-0.002 (0.011)	0.001 (0.011)	0.016 (0.011)
Survey-round FE	Yes	Yes	Yes
Wage-quintile FE		Yes	Yes
Age, race, educ., region FE			Yes
Mean of dep. variable	0.102	0.102	0.102
Observations	12,918	12,918	12,750

Note: Linear probability models of an indicator for job search in the past three months among currently employed men aged 25–55 in the NLSY97, 2008–2023 (Appendix D.2). The omitted category is the top liquid-wealth quintile; quintiles are computed within survey round using panel weights. Regressions are weighted by panel weights, and standard errors clustered by individual are in parentheses. *, **, and *** denote significance at the 10, 5, and 1 percent levels.

Table 7 reports linear probability models of the search indicator on liquid-wealth quintile dummies, with the top quintile as the omitted category. Column (1) controls only for survey-round fixed effects, column (2) adds wage-quintile fixed effects, and column (3) adds age, race, education, and census-region fixed effects.

In the full specification, workers in the bottom liquid-wealth quintile are 2.9 p.p. more likely to be searching on the job than observably similar workers in the top quintile with the same wage-quintile position—a 28% difference relative to the sample mean of 10.2%. The gradient is concentrated at the bottom of the wealth distribution: the two poorest quintiles search significantly more than the richest, while the middle quintiles are not sharply distinguished from the top. The profile is thus not quintile-by-quintile monotone, but the robust pattern is that search is more active at the bottom of the wealth distribution than at the top conditional on the wage, and it matches the model’s prediction. If anything, the conditional gradient in the data is steeper than in the model. Comparing workers at the same wage, the bottom wealth quintile in the model searches about 10% more than the top quintile at the bottom rungs of the ladder (Figure 13), against roughly 30% more in column

(3).

F Identification of Search Cost Function

As described in Section 4, [Faberman et al. \(2022\)](#) use the number of job applications or the hours of job search activity as a direct measure of job search, estimate the reduced-form elasticity of job search intensity with respect to residualized wages $\varepsilon_{s,w}^F$, and use it as a target for estimating the curvature of their search cost function.

To see the mapping between their model and my model, let e^F denote observed search intensity in [Faberman et al. \(2022\)](#). It is either the number of applications or the hours of job search in their model. Their offer-arrival technology takes the following form:

$$\lambda^F(e^F) = \alpha^F + \beta^F e^F, \quad (84)$$

where α^F captures unsolicited offers and β^F maps observed applications into offer arrivals. In my model, workers choose total search intensity s , which I write as $s = \underline{s} + e$, where $\underline{s}$ is the free-search component calibrated to unsolicited offers and $e \geq 0$ is costly search intensity. The contact rate is $\lambda(\underline{s} + e)$. If the model unit of costly search intensity e and observed applications e^F differ only by a scale factor $e = \kappa e^F$, then the contact rate is $\lambda \underline{s} + \lambda \kappa e^F$. Thus, relative to [Faberman et al. \(2022\)](#), my normalization corresponds to absorbing the slope of the offer-arrival function into the unit of search intensity and the aggregate contact rate. The intercept is captured by the free-search component $\underline{s}$. In particular,

$$\alpha^F \longleftrightarrow \lambda \underline{s}, \quad \beta^F \longleftrightarrow \lambda \kappa. \quad (85)$$

The scale κ is not separately identified from the aggregate search friction. However, the elasticity of costly search intensity with respect to wages is invariant to this unit normalization:

$$\frac{\partial e / \partial \log w}{\mathbb{E}[e]} = \frac{\partial (\kappa e^F) / \partial \log w}{\mathbb{E}[\kappa e^F]} = \underbrace{\frac{\partial e^F / \partial \log w}{\mathbb{E}[e^F]}}_{=\varepsilon_{s,w}^F}. \quad (86)$$

Therefore, although the level of search intensity is measured in model units, the reduced-form elasticity of costly search intensity is directly comparable to the empirical elasticity estimated by [Faberman et al. \(2022\)](#). I accordingly discipline the curvature of the search-cost function by matching the model-implied search elasticity to $\varepsilon_{s,w}^F$. Specifically, I target $\varepsilon_{s,w}^F = -0.36$,

the application-based elasticity in [Faberman et al. \(2022\)](#). The free-search component $\underline{s}$ is set so that the monthly rate of unsolicited offers equals 0.026 based on [Faberman et al. \(2022\)](#).

Computing the model-side elasticity. I compute the model counterpart of $\varepsilon_{s,w}^F$ as the coefficient of a linear regression of costly search intensity $e(a, w) = s_e(a, w) - \underline{s}$ on the log wage over the stationary distribution of employed workers, scaled by the average costly search intensity,

$$\varepsilon_{s,w}^M = \frac{\text{Cov}(e(a, w), \log w^{obs})}{\text{Var}(\log w^{obs}) \mathbb{E}[e(a, w)]}, \quad (87)$$

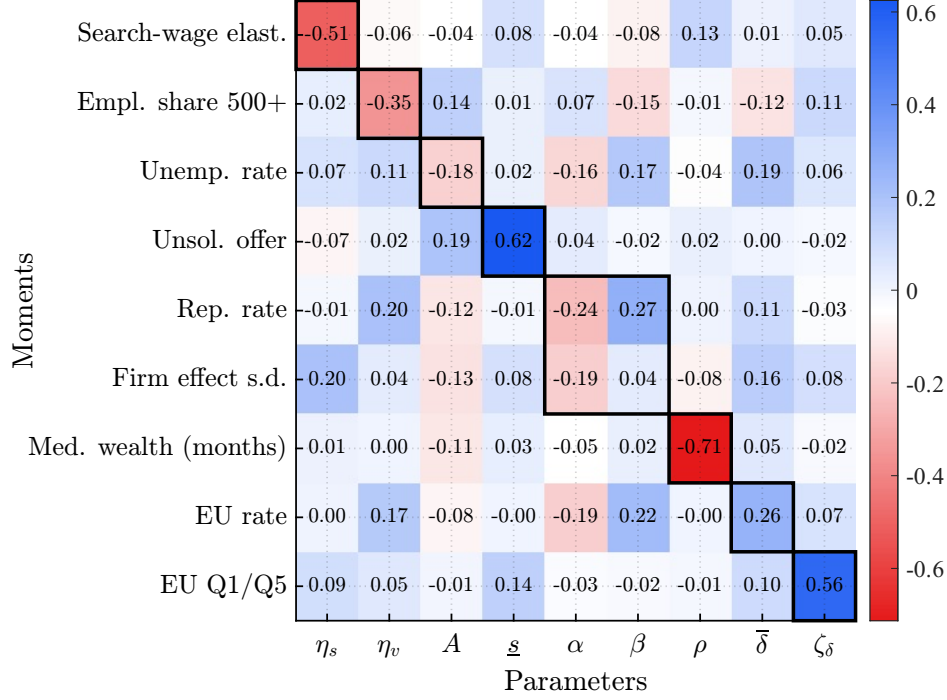
which replicates the construction of their application-based estimate — a level-on-log regression coefficient divided by the average number of applications. Computing the regression in levels of e keeps the zeros of workers near the top of the ladder in the moment, mirroring the zeros in observed application counts.

The regressor $\log w^{obs}$ requires care, which is the reason I do not simply use the model log wage. [Faberman et al. \(2022\)](#) estimate the elasticity with respect to *residualized* wages in the SCE, and their own calibration documents that most of the residualized wage dispersion reflects components other than the job ladder. Specifically, they attribute a standard deviation of about 0.24 to the firm (ladder) component and 0.57 to unobserved worker heterogeneity, with measurement error contributing a further 13% of the observed variance, for a total observed dispersion of $\sigma_w^{obs} = 0.66$. My model contains only the ladder component, with a cross-sectional standard deviation of log wages of 0.20. Regressing model search intensity on the model log wage alone would therefore compare a ladder-only elasticity in the model with an elasticity estimated on mostly non-ladder wage variation in the data.

To apply the same measurement structure to the model, I construct the observed log wage as $\log w^{obs} = \log w + u$, where u is classical noise with variance σ_u^2 chosen so that $\text{Var}(\log w^{obs}) = (\sigma_w^{obs})^2$. The noise represents the components of residualized wages to which search intensity does not respond, namely measurement error by definition and unobserved worker heterogeneity that is absent from the model counterpart of the regression in [Faberman et al. \(2022\)](#). Since u is classical and independent of search behavior, it leaves $\text{Cov}(e, \log w^{obs}) = \text{Cov}(e, \log w)$ and $\mathbb{E}[e]$ unchanged and only inflates the variance of the regressor, and the moment is computed analytically with no simulation:

$$\varepsilon_{s,w}^M = \frac{\text{Cov}(e(a, w), \log w)}{(\sigma_w^{obs})^2 \mathbb{E}[e(a, w)]}. \quad (88)$$

Figure 14: Absolute-Value Sensitivity Matrix



Note: Each entry reports the response of a targeted moment (rows) to a one-percent increase in a parameter (columns), converted to absolute values and normalized by the Sinkhorn–Knopp algorithm so that every row and every column sums to one. The signs of the underlying responses are restored in the reported numbers and the color scale. Black boxes mark the moment intended to identify each parameter. The single two-by-two box covering the replacement rate and the firm-effect dispersion indicates that the productivity-distribution parameters (α, β) are jointly identified by these two moments.

Equivalently, the ladder-only elasticity is attenuated by the factor $\text{Var}(\log w)/(\sigma_w^{obs})^2$, the classical errors-in-variables attenuation factor. Omitting this adjustment would ask the curvature parameter η_s to reconcile a ladder-only regression in the model with an estimate obtained on far more dispersed observed wages, and would push the estimated search cost function toward being implausibly inelastic.

G Identification of Model Parameters

Jacobian matrix. To clarify how individual parameters influence the objects matched in the calibration, Figure 14 reports a normalized sensitivity matrix obtained from the model’s Jacobian. Starting from the estimated parameter vector, I perturb each of the nine parameters by one percent and recompute all targeted moments. The resulting Jacobian, whose (i, j) element measures the percentage response of moment i to a marginal change in parameter j , is then converted into a nonnegative matrix by taking absolute values. To remove

differences in scale across both rows and columns, I apply the Sinkhorn–Knopp algorithm so that each row and each column sums to one, following Engbom et al. (2025). This normalization removes level differences across moments and parameters and makes the figure informative about their *relative*, rather than absolute, influence.

The normalized matrix highlights the relative importance of each parameter for each moment. If a moment were driven almost exclusively by one parameter, the diagonal entry would be close to one. In contrast, if all moments were equally informative about all parameters, entries would be more evenly spread across the columns. The boxed elements of the matrix in Figure 14 confirm that each targeted moment is strongly influenced by the parameter it is intended to identify. Specifically, for the first four moments, the diagonal element is the largest entry in its row, suggesting that these moments are helpful in identifying the corresponding parameters. The single two-by-two box covering the replacement rate and the firm-effect dispersion reflects that the two productivity-distribution parameters (α, β) are intended to be *jointly* identified by these two moments, and the relevant object is the block rather than its individual diagonal entries. The remaining moments are influenced by multiple parameters, but no two parameters display highly similar sensitivity profiles across the targeted moments. This lack of similarity indicates that each parameter affects the moments in a distinct way, helping the identification of parameters.

H The Perfectly Patient Firm Approximation

The firm block in the main text takes the $\rho_f \rightarrow 0$ limit as established in Section 2.3.3. This appendix solves the model without the approximation: the firm block discounts match profits at $\rho_f > 0$, tracking the new hire’s wealth path over the duration of the match. I conduct two exercises at the estimated parameters.

The first exercise is a nesting test. Setting $\rho_f \rightarrow 0$ in the non-approximated block must reproduce the benchmark, since Lemma 2.1 implies $Q(w) D(w; 0) = Q(w)/T(w)$. This holds: every calibration and untargeted moment agrees with the benchmark within 0.2%, and the pointwise identity holds with a median relative deviation of 0.03% across the productivity grid, exceeding 1% only in the bottom five percentiles of rungs next to the marginal firm, whose employment is negligible.

The second exercise is the robustness check referenced in the main text. I set $\rho_f = \rho \approx 0.05$, so that firms discount the future at the same rate as workers, and re-solve the equilibrium. Table 8 compares the resulting moments with the benchmark. All moments

Table 8: Equilibrium Moments with Firm Discounting

	Benchmark ($\rho_f \rightarrow 0$)	Firm discounting ($\rho_f = \rho$)	Difference (%)
Unemployment rate	0.060	0.062	+3.0
Search-wage elasticity	-0.360	-0.351	-2.4
Std. dev. log wage	0.199	0.195	-2.3
Empl. share 500+	0.519	0.494	-4.7
Unsolicited offer	0.026	0.026	+0.1
Replacement rate	0.403	0.412	+2.3
Med. wealth (months)	4.347	4.207	-3.2
EU rate (monthly)	0.015	0.015	+3.3
EU ratio Q1/Q5	5.291	5.188	-1.9
Monthly JJ rate	0.021	0.021	+1.6

Note: The nine targeted moments and the untargeted monthly job-to-job rate in the benchmark economy, where matches are priced by the steady-state formulation $Q(w)/T(w)$ ($\rho_f \rightarrow 0$), and in the non-approximated economy, where firms discount at $\rho_f = \rho$ and each match value is given by (20). The last column reports the percent difference.

change by less than 5%, and most by less than 3%. The largest change is in the employment share of large firms (−4.7%), reflecting that discounting slightly compresses the value of the long-duration matches of high-productivity firms, where turnover is lowest. The reason the approximation is innocuous is that the discount rate is an order of magnitude smaller than the average annual turnover rate, and discounting contributes little to the effective rate $\rho_f + T(w)$ at which match profits decay. Finally, the value of the approximation is analytical rather than computational. The non-approximated model solves in comparable time, but its first-order conditions involve the discounted duration $D(w; \rho_f)$, which has no closed form, whereas the $\rho_f \rightarrow 0$ limit collapses the match value into the static object $Q(w)/T(w)$ and thereby ties wage setting to observable turnover.

I Constant Search Benchmark

This appendix reports counterfactual economies in which search effort is not a choice, complementing the perfect-insurance benchmark of Section 5. The perfect-insurance economy removes only the wealth-dependence of search while keeping the search margin active; the economies here remove the margin itself. Comparing the two experiments separates the role

of wealth-driven search heterogeneity from the broader role of endogenous effort.

Design. Search effort is dropped as a choice and fixed at a constant level, and the matching efficiency A is recalibrated so that the average contact rate equals its baseline value; risk aversion, saving, and all remaining parameters are unchanged. I report two variants. In the *one-tier* economy, every worker searches at the baseline average effort ($s = 0.47$). In the *two-tier* economy, unemployed and employed workers search at their respective baseline group averages ($s_u = 1.17$ and $s_e = 0.42$), which preserves the average unemployed–employed effort gap and removes only the *within*-group heterogeneity — the wealth-driven margin at the center of the paper. Job acceptance remains mechanical and rank-based as in the baseline, so the counterfactuals operate only through the fixed effort levels.²³

Results. Table 9 reports the labor market outcomes, and Figure 15 displays the equilibrium schedules underlying them. The one-tier economy conflates two margins. The unemployment rate nearly triples (0.06 to 0.17) because unemployed workers lose their high search effort, and the wage structure moves at the same time: the average wage falls from 0.99 to 0.89, and the standard deviation of log wages rises from 0.20 to 0.26.

The two-tier economy separates the two margins. Restoring the average unemployed–employed effort gap brings the unemployment rate most of the way back (0.08 against 0.06 in the baseline), but the wage structure does not recover: the average wage falls further to 0.86, the standard deviation of log wages remains elevated at 0.27, and the average reemployment wage falls from 0.66 to 0.60. The monthly job-to-job rate declines from 0.021 to 0.016, and average labor productivity falls from 1.18 to 1.10, as slower reallocation shifts the composition of employment down the ladder. The comparison thus delivers a two-margin decomposition of what endogenous search contributes to the equilibrium. The unemployment consequences operate through the unemployed–employed effort gap, while the wage-structure consequences operate through within-group heterogeneity in search, which is precisely the wealth-driven heterogeneity that the perfect-insurance benchmark of Section 5 removes. Figure 15 shows the firm-side counterpart, by decile of the baseline employment-weighted productivity distribution: without the mechanism, turnover at the bottom rungs drops by nearly half while remaining unchanged higher up, the vacancy yield falls and its gradient across the ladder flattens, the wage schedule shifts down through the interior of

²³One by-design feature of the two-tier economy is disclosed for transparency. With unemployed effort fixed at the high group average, unemployment is worth more than the lowest rungs of the ladder for 3.8 percent of workers, who lose search capacity upon employment. Because acceptance is mechanical, this inversion affects values, not flows.

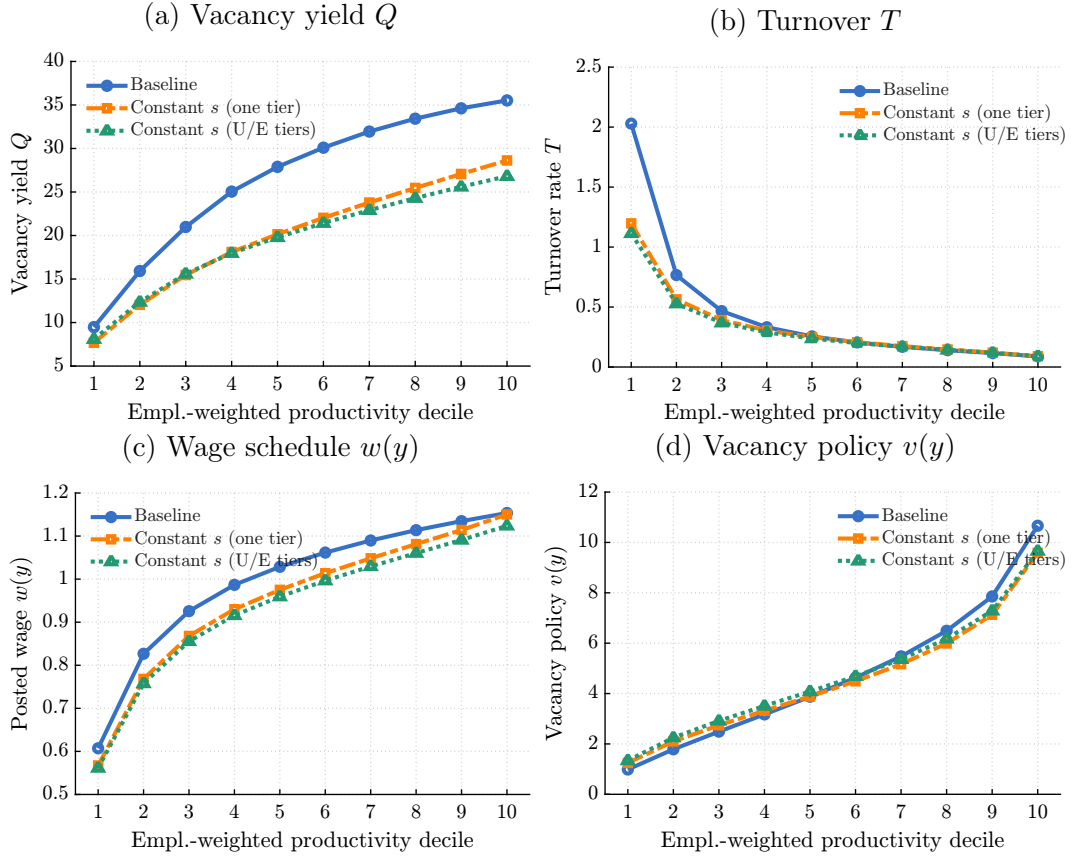
Table 9: Labor Market Outcomes without Endogenous Search

	Baseline	Constant s (one tier)	Constant s (U/E tiers)
Std. dev. log wage	0.20	0.26	0.27
Avg. wage	0.99	0.89	0.86
Avg. reemployment wage	0.66	0.61	0.60
Unemployment rate	0.06	0.17	0.08
JJ rate (monthly)	0.021	0.017	0.016
Avg. labor productivity	1.18	1.12	1.10

Note: Levels of the same outcomes as in Table 5. The one-tier economy fixes every worker's search effort at the baseline average; the two-tier economy fixes unemployed and employed effort at their respective baseline group averages. In both, the matching efficiency is recalibrated so that the average contact rate equals its baseline value, and all other parameters are unchanged.

the ladder, and the vacancy policy tilts modestly toward low-productivity firms as the wage discipline at the bottom weakens.

Figure 15: Equilibrium Schedules without Endogenous Search



Note: Equilibrium schedules in the baseline and in the two constant-search economies, by decile of the baseline employment-weighted productivity distribution. Decile values are employment-weighted averages using baseline weights in all three economies.

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